2. LINEAR MOTION

You will study the motion of an object in one dimension from a number of points of view. You will demonstrate how the variables of motion are related by differentiation and integration and investigate the relationship between potential and kinetic energy.

Theory

Why Study Motion?
Motion is everywhere in the universe. Only at a temperature of absolute zero is the motion in any body truly absent. If motion exists then so also does energy. To the delight of the physicist the tools that were invented by Galileo, Newton and others 200 years ago to describe motion apply everywhere in the known universe, from electrons in our own bodies to the farthest galaxy. The study of motion and of energy is at the heart of physics. This experiment deals with the simplest kind of motion, that in one dimension.

Kinematics and Dynamics
The subject of motion is divided into kinematics and dynamics. Kinematics is concerned with the aspects of motion which exclude the forces which cause motion; thus, in a manner of speaking, kinematics is focussed on the development of definitions: position, displacement, velocity, acceleration and on the relationships between them. Dynamics widens the study of motion to include force and energy.

Definitions
Position
Kinematics begins with the idea of position. Suppose that we photograph an object moving to the left along a horizontal path at two instants of time and superimpose the images for study (Figure 1). We examine one image with a ruler and mark off the number of units which separates the object from the ruler's zero. The zero is a reference or origin at a position of zero units by definition while the object is at another position, say x units. x is an instantaneous quantity since it applies to a specific clock time—the instant of the taking of the photograph. Position like length is a basic quantity being dependent only on the unit adopted.

But position involves also direction: in principle the object could be to our right or to our left. To include the information of direction we use a vector. The magnitude or length of the vector, say \( \hat{r} \), is x (or \( r \)), while the direction is to the right, meaning the object is to the right of the reference point. We could also agree that, by convention, the sign of x is to be positive here.

Elapsed Time
The two positions of the object in Figure 1 must be labelled with different vectors and different clock times. The photographs can be said to show two events, an initial “i” event and a final “f” event. There is now an elapsed time between the events given by:

\[ \Delta t = t_f - t_i \] …[1]

(unit seconds, abbreviated s). Keep in mind the difference between the two concepts of time; an elapsed time is the difference between two clock times.
2. Linear Motion

Figure 1. Shown here is an object moving toward the origin (left) at two positions with the corresponding clock times indicated. Position, displacement and velocity vectors are given different head styles to emphasize their different nature.

Displacement

Displacement differs from position. In the interval of time between the events the object moves from one position to another. The displacement is the difference between the two vectors describing the two positions

\[ \Delta \vec{r} = \vec{r}_f - \vec{r}_i \]  \[ \text{[2]} \]

(unit meters, abbreviated m). Displacement, being the difference between two vectors, is a vector. The displacement is negative (according to our convention) since it points towards the origin.

Velocity

Another quantity in kinematics is the average velocity, or the displacement an object undergoes in one second of elapsed time. This is the ratio

\[ \frac{\Delta \vec{r}}{\Delta t} = \vec{v} \]  \[ \text{[3]} \]

(unit meters per second, abbreviated m.s\(^{-1}\)). The average velocity, being a vector divided by a scalar, is a vector. The average velocity is negative here, too, since it points towards the origin; its magnitude is the speed.

The elapsed time in eqs[1] and [3] is a finite interval. What if it is infinitely small? Mathematically, this amounts to taking the limit of eq[3] as \( \Delta t \to 0 \). The increments \( \Delta \) are replaced by the differentials \( d \). Eq[3] then becomes what is known as the instantaneous velocity

\[ \frac{d\vec{r}}{dt} = \vec{v} \]  \[ \text{[4]} \]

This quantity is abstract and tricky to imagine: it can be thought of as the average velocity that might be measured (with a superior detection system) over an infinitely short elapsed time (or the velocity at a specific clock time). In practice, it can be obtained only approximately.

If the displacement is known as an analytical function of time, \( r(t) \), then the instantaneous velocity at some clock time \( t_0 \) is the tangent to the function at \( t_0 \), or the first derivative of \( r(t) \) at \( t_0 \). The finding of tangents is one of the objectives of this experiment.

Acceleration

The velocity of the object in Figure 1 may change with time. The velocity could decrease as a result of a force of friction with the path. Or the velocity could increase if the path were not horizontal and a component of the force of gravity acts on the object. The time rate of change of the average velocity is called the average acceleration and the time rate of change of the instantaneous velocity is called the instantaneous acceleration. Both types of acceleration are defined as in eqs[3] and [4] with "v" substituted for "r" and "a" substituted for "v".
Motion of an Object Whose Velocity is Constant

In this experiment you will mostly be concerned with the motion of an object whose velocity is changing. However, for purposes of understanding we first consider the case of constant velocity.

The case of an object moving towards the origin on a horizontal plane is drawn in Figure 2. We suppose that the data pairs \((t, r)\), where \(t\) is the clock time and \(r\) is the position are measurable at regular intervals by some detection system. Two such points when plotted on a graph might appear as shown in the upper half of Figure 3.

A computer could be programmed to calculate the “average velocity” as the slope between the two datapoints and plot it as a point on a graph (lower half of Figure 3). The result is negative, the sign indicating the direction of the velocity vector.

The computer software used in this experiment does something similar by finding the average velocity by averaging over the slopes between a number of datapairs (7 by default). Thus if a number of datapoints were measured and the results plotted on a graph, the result might resemble Figure 4.

As the glider approaches the origin here the position decreases but is always positive; while the velocity remains at a constant negative value. The velocity is therefore just the derivative or the slope of the displacement versus clock time graph (or the slope of the position versus clock time graph here in one dimension). The velocity is seen to change little (if at all) with clock time and so the acceleration (deceleration) is very small.

![Diagram](image)

Figure 2. An object is shown at two positions (events) while moving toward a detector on a horizontal plane.

![Diagram](image)

Figure 3. A graph of the two position-clock time datapoints described in Figure 2. Shown also is a point on the velocity graph as it might be generated from the slope between the two datapoints multiplied by the sign of the velocity vector.
Motion of an Object Whose Velocity Changes with Time

In this experiment you will mostly be ignoring the effects of the force of friction. However, for purposes of understanding it is useful to consider friction, however briefly.

We know that a small force of friction exists between the glider and the layer of air on which it moves. Friction acts opposite to the direction of motion (to the right in Figure 2) and therefore produces an acceleration also toward the right. This acceleration is often described as a deceleration in that it is opposite to the velocity and describes a velocity decrease. (The object is slowing down.) The velocity and acceleration versus clock time graphs in this case will resemble Figure 5.

It is known from other experiments (“Simple Measurements”) that the force of friction, though small, has a complicated functional form giving rise to a deceleration which depends on the first (and sometimes the second) power of the velocity. Gravity, unlike friction, is a constant force and is therefore much easier to deal with; the effect of gravity on motion we consider in the next section.

Motion of an Object Whose Acceleration is Constant and Nonzero

Figure 6 shows an object moving with constant non-zero acceleration down an inclined plane (neglecting friction). Its free body diagram is drawn in Figure 6b. Starting from the assumption that the acceleration is constant and nonzero we can develop the mathematics of the object’s motion using successive integrations. If friction is negligible, the mag-

Figure 4. Typical position and velocity graphs as might be produced for an object moving as shown in Figure 2.

Figure 5. Velocity and acceleration graphs for an object moving as shown in Figure 2 while subject to a small force of friction. Remember, graphed here are the magnitudes of the vectors multiplied by the sign corresponding to the direction of the vectors.
The glider’s instantaneous speed at some arbitrary clock time $t$ is obtained by integrating $a$ from eq[5]:

$$v(t) = \int a \, dt = at + C_1,$$

where $C_1$ is a constant of integration. $C_1$ is the instantaneous speed at a clock time of 0 seconds, i.e., the quantity $v(0)$; thus

$$v(t) = v(0) + at. \quad \text{...[7]}$$

The glider’s position at the clock time $t$ is found by integrating $v(t)$ from eq[7]:

$$r(t) = \int v(t) \, dt, \quad \text{...[8]}$$

$$= \int (at + v(0)) \, dt,$$

$$= \frac{1}{2} at^2 + v(0)t + C_2,$$

where $C_2$ is another constant of integration. $C_2 = r(0)$, the position at a clock time of 0 seconds; thus

$$r(t) - r(0) = \frac{1}{2} at^2 + v(0)t. \quad \text{...[9]}$$

We can now make the following correspondence. At event $i$ we let $t_i = 0$, then $r_i = r(0)$ and $v_i = v(0)$. Then we can also write for the event $f$, $t_f = t$, $r_f = r(t_f)$ and $v_f = v(t_f)$. Then equations [9] and [7] take on the more familiar forms:

$$r_f - r_i = \frac{1}{2} at^2 - v_it \quad \text{...[10]}$$

and

$$v_f = v_i + at. \quad \text{...[11]}$$

You can test these equations with the detection system and software in this experiment several ways. Firstly, you can test if the position, velocity and acceleration data you collect really is described by functions of the form of eqs[10] and [11]. In other words, you can perform polynomial “fits” to the data, finding in the process the value of “$a$”. You can test the differential and integral relationships between the variables by invoking the options of slope and integral. These are the objectives of this experiment.

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Figure 6. (a) A object is shown at two positions while moving down an inclined plane. The object’s free body diagram is shown in (b). The object is “pulled” toward the origin by the component of the force of gravity $mg \sin \theta$. The acceleration is negative in this case according to the sign convention employed by the detection system.
In Figure 6 the object is moving down an inclined plane and is therefore accelerating. Any time an object is accelerating, work is being done upon it. The concepts of force and energy take us into the area of dynamics.

The concepts of work and energy are closely related. By definition the work done on a point mass $m$ by a net force $\vec{F}$ is

$$W = \vec{F} \cdot \vec{r} = Fr\cos\theta, \quad \text{...[13]}$$

(unit joules, abbreviated J). Here $\vec{r}$ is the object’s displacement vector and $\theta$ is the angle between the vectors $\vec{F}$ and $\vec{r}$. (The $\cdot$ symbol indicates the vector dot product.) According to the Work-Energy Theorem the work done on an object goes to increase the object’s mechanical energy and this energy can be either kinetic energy, potential energy, or both.

Both types of energy can be defined with reference to Figures 7 where an object of mass $m$ is shown responding to an applied force in two ways. In Figure 7a the object is being moved to the left in our field of view along a horizontal, frictionless surface. In Figure 7b the object is being moved vertically upwards. Figure 7a enables us to define kinetic energy most easily, Figure 7b potential energy. Let us first consider the former.

![Figure 7](image)

**Figure 7.** An object is shown responding to a force in two ways. In (a) the surface is horizontal and frictionless, so the work done by $\vec{F}$ goes entirely into increasing the object’s kinetic energy. In (b) $\vec{F}$ is greater then the weight $\overrightarrow{F_g}$ of the object by some infinitesimal amount so the work goes entirely into increasing the object’s potential energy.

**Kinetic Energy**

Suppose (Figure 7a) that we are able to measure the instantaneous speed of the object at the clock times $t_i$ and $t_f$ when the object is at positions $r_i$ and $r_f$ respectively. The object is receiving a force $\vec{F}$ and is being accelerated. The work done on the object between $r_i$ and $r_f$ is, from the definition, eq[13]$^3$

$$W = \int \vec{F} \cdot d\vec{r} = m \int \vec{a} \cdot d\vec{r} = m \int adr,$$
since the vectors \( \vec{a} \) and \( \vec{dr} \) are parallel. Changing the variable of integration from \( r \) to \( v \) gives

\[
W = m \int \left( \frac{dv}{dt} \frac{dr}{dt} \right) dt
\]

\[
= m \int_{v_i}^{v_f} v dv
\]

\[
= m \left( \frac{1}{2} v_f^2 - \frac{1}{2} v_i^2 \right).
\]

The work done is therefore the difference between two terms:

\[
W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2. \tag{14}
\]

If we call the expression \( m v^2/2 \) the kinetic energy \( E_k \) or the energy of motion, eq(14) can be written

\[
W = E_{kf} - E_{ki} = \Delta E_k. \tag{15}
\]

In this particular case \( W \) is a positive number, i.e., the kinetic energy of the object is increasing. The work done on the object is therefore equal to this increase in kinetic energy. This is one-half of the Work-Energy Theorem.

### Potential Energy

If we assume in Figure 7b that the applied force \( \vec{F} \) is insignificantly greater than the weight of the object \( F_g \) acting downwards then the object moves upwards a distance \( \Delta h = h_f - h_i \) in some elapsed time at zero speed (thus acquiring negligible kinetic energy). From the definition, eq(13), the work done on the object is

\[
W = \int \vec{F} \cdot d\vec{r}
\]

\[
= \int_{h_i}^{h_f} mgdr.
\]

Once again the work done is equal to the difference of two terms:

\[
W = mgh_f - mgh_i. \tag{16}
\]

If we call the expression \( mgh \) the potential energy of the object at the position \( h \) relative to the position 0 and denote it \( E_p \) we can write eq(16) as

\[
W = E_{pf} - E_{pi} = \Delta E_p. \tag{17}
\]

This is also a positive number. Thus the work done on the object is equal to the increase in the object’s potential energy. This is the second half of the Work-Energy Theorem. There is no change in kinetic energy here. This energy is potential energy, gravitational potential energy to be exact. This is the energy an object possesses by virtue of its position relative to some reference position in a gravitational field. This energy is the work required (by us) to move the object (very slowly) to the final position from the initial, reference position.

Now Figures 7 describe two idealized situations where the object is moving either horizontally or vertically. Should the object be moving on an inclined plane it will undergo changes in both kinetic and potential energy. One of the objects of this experiment is to study this general case.
Exercise 0. Preparation

Orientation
Identify the glider stowed on top of your air blower. Find the spacer. When the time is right you will place this spacer beneath the front footpad of the track to form an inclined plane. Identify the detector (the blue box) mounted just beyond the near end of the track) and the universal interface (ULI) board mounted on the front of the blower box.

Simple Measurements
You are going to be logging the glider’s mechanical energy so you need its mass. You can weigh it on one of the triple beam balances in the lab. You can measure the thickness of the spacer with a vernier caliper. (You need to record the experimental uncertainty in both measurements, of course.)

The Air Track
The object whose linear motion you will study is a glider on an air track (Figure 8). An air track is an ideal device for studying the motion of an object like a glider since it supplies a thin film of air on which a glider, having just the right shape, can move with reduced friction. Identify the air track’s two rear feet and its single front foot. Confirm that elastic bands are in place to serve as bumpers for the glider.

The detector and associated electronics log the glider’s position at clock times you can select. The detector emits pulses of ultrasound at a frequency of 50 kHz. These pulses travel out from the detector through the air, reflect from the reflector on the glider and return. Since the speed of ultrasound in air at room temperature is known, the computer calculates the distance of the glider from the detector (the position) by timing the round trip of the pulse. To get velocity and acceleration, the computer makes several measurements and calculates averages and rates of change.

Figure 8. The air track when inclined (scale is exaggerated here). X is the distance between the single front foot and the midpoint of the two rear feet. For all tracks X = (2.29 ± 0.01) m.
Checks to Minimize Friction
Check the following to insure that your glider moves with minimum friction:

✓ If ON, turn any and all equipment OFF.
✓ Your glider must match the track or it will not run properly. Read and follow the instructions on the blower box.
✓ Turn the air blower ON.
✓ The pressure of the air must be sufficient to support your glider. Read and follow the instructions on the blower box.

Level Check
Your air track must be level when you start this experiment otherwise your glider will experience unaccounted for acceleration. To check for level first remove any spacer that might have been left under the front footpad of the track. Place your glider on the midpoint of the track and look for any drifting right or left. If your glider does drift adjust the screw on the front foot of the track to compensate. When the track is level your glider should not drift appreciably left or right.

Setting up the ULI
✓ Turn the ULI board ON by means of the switch located in its lower right hand corner. A LED on the board should glow red or green.

Booting LoggerPro
To boot the program follow these steps:

1. With the computer ON log into your account on the college network as you learned to do at the Orientation Workshop. (If for any reason you cannot log on you can save your work temporarily in the “Student Temp Save” folder on the local hard drive. In any case, you can log in and out at any time.)
2. On the local harddrive “Macintosh HD”, locate and open folders in this order: “Physics”>> “PHYA10/A20F” >> “02.Linear Motion”.
3. Inside “02.Linear Motion” double click the icon Linear Motion. LoggerPro and the Linear Motion setup will run.

The Opening Screen
The opening screen shows three narrow graph areas (one of which is shown in Figure 10). The position versus clock time graph should be currently selected (a box should be drawn around it). To select any graph move the mouse to position the cross-hair pointer over the graph and then click the mouse button once. Notice on the clock time axis of the graphs that the program is set by default to record for 10 seconds.

First Options
LoggerPro has a number of options, many of which you will encounter in due course. Two options you can examine before collecting data are DataRate and Averaging. (But if you wish to bypass you can go directly to The Expected Value of Acceleration.)

Option 1. Datarate
The number of datapairs \( (t, r) \) LoggerPro records per second is set by the data rate. To see the current data rate choose Experiment >> Sampling. If the program runs for 10 seconds at the current data rate (samples/second) how many datapoints will be collected? Look at the data rate now but leave it unchanged.

Option 2. Averaging
LoggerPro finds velocity and acceleration by a process of averaging. To examine averaging choose Experiment >> Options. From the information you see, how many datapoints are averaged in computing a derivative? Look at the averaging now but leave it unchanged.

The Expected Value of Acceleration
Locate the spacer you will be placing under the front footpad of your track. Based on the geometry of Figure 8 what value of acceleration should you expect for your glider? You need this number in the formula for potential energy (next section).

Entering Formulas
LoggerPro automatically computes velocity and acceleration from the basic values of position it records, but it must be instructed to calculate energy (since energy depends on your glider’s mass).
You have to enter the formulas into LoggerPro’s spreadsheet. To do this select Data >> New Column >> Formula. Figure 9 shows a fragment of the screen which then appears. On separate calls to New Column enter formulas for kinetic energy, potential energy and total energy as is shown in the figure. Pay attention to the common spreadsheet syntax (”*” means multiply, “^” means to the power of, etc.) Notice that you can select variables and functions from the popdown lists. You enter velocity as “Velocity” which you can either type in yourself or select from the Variables popdown list. Take your time here. Your TA will help you with these tasks.

The formula you will likely have the most difficulty with is potential energy. Convince yourself the formula can be written

$$\Delta E_p = m \times (\text{expected value of } a) \times \text{position}.$$ 

Or, in LoggerPro notation:

$$m \times (\text{expected value of } a) \times \text{“Position”}.$$ 

What is the expected value of “a” based on a 2 cm spacer?

If later, you find you have made a mistake in typing and wish to modify or remove the column go back to Data >> Modify Column and correct your mistake. When ready click OK.

**Changing the Graph in an Area**

By the time you have finished entering these formulas you may observe that the label down the left hand side of the graph has changed (look ahead to the left hand side of Figure 10). You can change what is displayed in a graph area. To do this place the pointer over the label and hold down the mouse button. You can then select the graph you wish from the list of Columns to Plot. Do this now to change the label on the first graph back to "r (m)".

Figure 9. A fragment of the screen which appears when you select New Column.
Exercise 1. First Launch, More Options and First Observations

The objective of this exercise is to obtain some data from a first launch, to explore a few options of the program and to examine the data qualitatively.

First Launch
After you have levelled your air track satisfactorily incline the track with the spacer. Now release (don’t push) the glider from the top end of the track. Make sure the glider’s reflector is facing the motion detector.

Record data between two bumps of the glider with the lower end of the track. (Though this is not critical it is convenient to begin the recording just before one bump occurs so that the recording ends just after the second bump occurs.) When ready click the Collect button. Repeat a few times to get graphs to your liking. After 10 seconds you should see screens resembling Figure 10. You should have 6 graphs of information altogether: Position, Velocity, Acceleration, Kinetic Energy, Potential Energy and Total Energy (the last three you have to select using Option 1 below). Before you get down to serious business you may wish to investigate one or more of the following options. However, if you wish to bypass the options you can go directly to the section First Observations.

Option 1. Changing the Graph in an Area
Notice in Figure 10 the label “r (m)”. You can change what is displayed in a graph area. To do this move the mouse to place the pointer over the label and hold down the mouse button. A list of Columns to Plot will then pop down. Select the graph you wish to be displayed in the area from this list.

Option 2. Number of Panes
You may wish to display one graph at a time or as many as four graphs. To change the number of panes select View >> Graph Layout…. Then click on the number of panes selection you wish.

Option 3. Saving
If you wish to save your experiment choose File >> Save As… . If you wish to just save the data choose File >> Export Data…. Make sure you save your file in your account space on the network or in the “Student Temp Save” folder on the local hard drive.

Option 4. Preparing for Printing
Eventually, you will want to print your graphs on the laserwriter printers in the physics lab as proof of your work so it is useful to prepare for printing.

Figure 10. A Position vs time graph. One of the six screens of information you have collected at this stage of the experiment.
now. Choose File >> Page Setup. Fill in your name and any comment you wish to have appear beneath your graphs on the printed page and click OK. Your partner may change this later to print graphs for his or herself.

**Option 5. Printing**

Before you attempt to print anything on the laserwriter printers in the physics lab, confirm that your output will go to the printer you expect. Select ⌘ > Chooser > LaserWriter 8, then confirm that the correct printer (PhysicsLaser-WriterI through PhysicsLaserWriterV) is selected. If in doubt about a printing issue ask your TA. To actually print a graph choose File >> Print Graphs.

**First Observations**

Before you apply mathematics to your data in any experiment it is often useful to obtain an overall qualitative “impression” of your data to determine if it is “reasonable”. To this end consider these questions:

Questions:

? With reference to eqs[10] and [11] do you have a graph which is quadratic in time? Linear in time? Which graph describes a constant (between bumps)?

? Your position versus clock time graph resembles the path of a projectile in two dimensional space. Why?

? Explain in your own words why the velocity and acceleration graphs have the form you observe.

? From your velocity versus clock time graph you should be able to find a clock time at which the velocity of the glider was zero. What was the acceleration of the glider at this clock time? If the velocity of a body is zero can its acceleration be non-zero? Explain.

With LoggerPro’s tools you can call up instantaneous values and tangents. To see instantaneous values choose Analyze >> Examine. One of the graphs will resemble Figure 11. If you now choose Analyze >> Tangent the tangent at the position of the cursor will be shown on the graph.

By means of the tangent function can you demonstrate to your satisfaction the validity of eq[4] and its counterpart for acceleration? A few tests are sufficient because you will continue this in Exercise 3. When you wish to turn Analyze OFF choose Analyze >> Examine a second time. A better test of the validity of eqs[10] and [11] is a polynomial fit. This you will do in the next exercise.

![Graph produced by LoggerPro, copied to the clipboard and pasted directly into Microsoft Word.](image)

**Figure 11.** One of the six graph screens with Examine turned on. As you move the cursor with the mouse the position and time are updated at the top left corner of the screen.
Exercise 2. Fitting Functions to Data

The objective of this exercise is to test the validity of eqs[10] and [11] by seeing if they can be fitted to the appropriate set of your data, position, velocity or acceleration. The result of the fitting will yield the acceleration “a”. This value you can then compare with the “expected value” of Exercises 0 and 1.

Fitting

To fit a function to your data first select (click on) the appropriate graph. Then choose Analyze >> Examine. Place the cursor at the beginning of the range you wish to fit, hold down the mouse button and drag over the range you wish fitted. Make sure the range you select does not include a bump (after all, you are interested in linear motion and a bump is definitely non-linear). If you wish to fit a straight line choose Analyze >> Linear Fit. If you wish a different kind of fit (a quadratic for example) choose Analyze >> CurveFit. The result of a linear fit might appear as shown in Figure 12.

For this process to succeed you must pay close attention to the function to be fitted and the terms to be included. For example, eq[11] is what kind of function of time? Linear? Quadratic? What is the slope? What is the intercept? If you wish more information on the linefit (more digits etc) double click on the bar of the information window. The result will resemble Figure 13.

Fit Results

Find the acceleration “a” from the results of the fit of all three graphs (position, velocity and acceleration). How do the "a" values compare with each other and with the expected value? Comment on the quality of the fits. If the fits are good it can be said that the equations “describe” the data.

Figure 12. A range of data on a velocity vs clock time graph has been selected (shown by the square brackets) and fitted to a straight line. The slope and intercept are, respectively, -0.090 and 0.512 (units to be supplied by you). To request more information about a fit double click on the bar of the fit information window (Figure 13).

Figure 13. The graph of Figure 12 showing more information about the linefit. The slope is now seen to be -0.0897 ± 0.0001. Cor is the correlation coefficient.
Exercise 3. Differentiation and Integration

The objective of this exercise is to investigate how the definitions of motion are related by differentiation and integration.

Differentiation (Slope Studies)
Identify the sections on the position versus clock time graph where the glider is moving towards and away from the origin. Explain in your own words why the position increases or decreases and when. Is the value obtained for the slope of the position versus clock time graph at some arbitrary clock time consistent in magnitude and sign with the value given for the velocity? Is the value obtained for the slope of the velocity versus clock time graph consistent in magnitude and sign with the value given for the acceleration? A few tests will suffice.

Integration (Area Studies)
Comparing two variables of motion related by differentiation—when one is the slope of the clock time graph of the other—is easy. But what if the relation is integration? To prepare ourselves we put eqs[9] and [10] in this form:

\[ r(t_2) - r(t_1) = \int_{t_1}^{t_2} v(t)dt \]  

where \( t_1 \) and \( t_2 \) are, of course, the clock time limits.

As preparation for the following describe eq[18] in your own words. What does its left hand side mean? What does its right hand side mean? You might wish to include in your word description the phrases “difference between” and “area under the curve of…” or other equivalent phrases. Pay attention to the clock time limits. If you understand this expression enough to explain it you will have no trouble with the following activities.

*LoggerPro* will compute the area under a region of a curve you select, i.e., between two limits. You must first select the region by dragging with the mouse as was illustrated in Figure 12. Pick a region of the velocity versus clock time graph between two bumps (which is linear) and drag over the region to select it. Next choose Analyze >> Integrate. Do your best to confirm eq[9]. Repeat in a similar fashion for the relationship between the velocity and acceleration.

Figure 14. A velocity vs clock time graph showing an integration. The area under the curve for the time limits chosen is 0.222 m/s/s.
Other Features and Other Questions
There are other features of the graphs whose explanation involves interesting physics. For example, explain in your own words and with sketches why the features shown in Figures 15a and 15b appear on the velocity and acceleration graphs. Why does the acceleration go from zero to a maximum positive value then drop to zero again? Explain why both “positive” and “negative” bumps appear on the acceleration graphs. Does the magnitude of the velocity have the same value after, as before, a bump? What does this say about the kind of collision at the bump?

![Figure 15. Interesting structures in the velocity and acceleration graphs.](image)

Exercise 4. Energy
The objective of this exercise is to examine as many aspects of energy as time permits.

Viewing the Energy Graphs
To view the energy graphs follow the procedure of Option 1 of Exercise 1. Your total energy graph may or may not resemble Figure 16.

Observations and Questions
Explain in your own words the characteristics of the kinetic energy graph. That is, answer or otherwise account for the following (interrelated) questions:

- What accounts for the “dips” in the kinetic energy graph?
- The dips in the kinetic energy graph show that the kinetic energy is zero for a brief instant. How is it possible for the glider to lose all of its kinetic energy and then regain it?
- Describe all instances of work being done and by what.

Conservation of Energy
Questions:
- Is energy conserved in this experiment? If not, why not?
- Why are the potential energy and kinetic energy graphs curves and not straight lines?

Energy Loss
Of course, friction is present here and does account (at least partially) for the decline in total energy. How would you calculate the total energy lost per second?

Other Questions
- What might be another source of energy loss besides friction?
- How would you go about quantifying this loss?
Figure 16. A total energy vs clock time graph. The energy is nearly a constant here at just under 0.05 J. It would still be useful to find the energy lost per second from a line fit.

Physics Demonstrations on LaserDisc

There are many physics demonstrations on laserdisc dealing with motion in one dimension. Here are just a few of them:

from Chapter 2 Linear Kinematics
   Demo 01-08 Constant Velocity
   Demo 01-11 Constant Acceleration
from Chapter 3 Linear Dynamics
   Demo 03-01 Air Track Friction

Activities Using Maple

E02Linear Motion
(Under Construction)

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End Notes for Linear Motion
1. If you can’t follow the calculus here go directly to the main equations [10] and [11].
2. By point mass is meant an object whose internal structure is to be neglected. Or in other words, a point mass is an idealized object of mass $m$ of infinitely small size. Some of the work done on an object of finite size could conceivably go into increasing the object’s internal energy, something we wish to avoid. In this guidesheet the word object will mean point mass.
3. If you wish you to bypass the calculus go directly to the main results, eqs [15] and [16].
4. This change is effected by putting $a = \frac{dv}{dt}$ and $dr = \frac{dr}{dt} dt$.
5. This is similar to how a bat sees in the dark using its own kind of ultrasound. It is also how a Polaroid autofocus camera determines the distance to an object to focus properly. Similar principles apply to ultrasound devices used in medicine and to various range-finding devices.