Uncertainty, Risk, and Market Formation

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In simple models of market formation, firms see larger markets as less risky. However, there is a diminishing marginal return to participating in larger markets. At the same time, in a simple spatial economy, larger markets mean the marginal participant in a larger market must incur an additional shipping cost. This is the stuff of the Economides-Siow model. That model, while interesting, raises questions about the nature and size of a firm and the firm's ability to internalize risk. In this paper, I redefine the nature of a firm using a cooperative to internalize risk. In general, economists think that competition is efficient. The Economides-Siow model ignores spatial equilibrium within a market. If some firms are closer to the market and others are further away, there will be an incentive for the firms at greater distance to want to relocate nearer the market. One solution is to assume that firms reach spatial equilibrium by forming a cooperative in which members share the aggregate cost of shipping equally. Another is to assume that firms reach spatial equilibrium by bidding up the price (rent) for land at advantageous locations. In support of my argument that the cooperative may be integral to firm design and efficiency, I show that the return to a firm in an Economides-Siow model is in general lower than the return where there is a cooperative (absent organizational cost).

1. The Economides-Siow Problem

Typically, locational models look at a market in terms of the constraint on participation posed by the existence of shipping costs. In the case of an expendable commodity, a customer is seen to participate in a market if the effective price (purchase price plus unit transaction cost) does not exceed the maximum price the customer would be willing to pay. A unit shipping cost—which increases with distance shipped—limits the geographic extent of a market to its range. However, we can cast the problem more generally. Why do local markets exist at various places across the landscape? In choosing to participate in a particular local market, a participant has more to gain than simply the ability to purchase a product. Presumably, to the extent that there is more than one possible market, participants might be attracted to one market over another because they think they are more likely to get a ‘good price’ there. Underlying this idea is the notion that markets are inherently uncertain. Otherwise, if market outcomes were known with certainty, why would not otherwise-identical consumers always choose the same market?

In this paper, I look at models of agricultural markets. In each of them, I imagine tenant farmers—as firms—produce a crop using rented land as an input and then ship all or part of that crop to a central place to exchange. I assume here that all land is equally fertile. I assume also that landlords maximize rents, are competitive (not collusive), and are numerous enough that each is a price taker in the local market for land. Such models can be used to describe a simple kind of regional economy. As each firm occupies geographic space, there is therefore a limit on the number of them that can be accommodated within a given land area. In later parts of this paper, I use that feature of the model in thinking about how large a market might be in terms of the number of market participants. To the farmer, there are costs to exchange: e.g., the cost of shipping produce to a local market and possibly purchases back home. Presumably, a farmer plans a particular mix of produce taking into account the farm’s preferences, available resources including skills of its workforce and fertility of soil. Regardless of its plans, the harvest reaped by the farm depends also on events such as climatic variations that are beyond the farm’s control. To maximize its utility, a farm whose harvest consists mainly of grain production might want to exchange with other farmers whose harvest was mainly other kinds of desirable produce: e.g., eggs, milk, meat, or vegetables. As a supplier, you decide the type and quantity of produce to

1 This paper is drawn from my forthcoming book, The Geography of Competition: Firms, Prices, and Localization, to be published by Springer.
bring to market without knowing in advance what will be brought by other farmers. As a consumer, in deciding whether to attend a local market, you cannot be sure in advance about the availability, quality, and exchange rate (price) of various produce. In the interest of simplicity, assume buyers and sellers have perfect information once they arrive at the local market and that this translates into a single Walrasian market-clearing exchange rate for the day; all those who want to exchange produce at that rate are able to do so. Under what conditions will a farm choose to participate in a particular local market? If a farm has a choice between participating in a local market nearby or a larger market further away, which will it choose, and why?

This paper is inspired by a pioneering spatial model of liquidity and market size in Economides & Siow (1988). At the same time, the (E&S) model is different from the one presented here. How? First, the E&S model assumes that actors maximize expected utility. Here, however, I assume behavior based on risk-return. A second difference from the E&S model is that I start with a version where geography plays no role. Later in the paper, I introduce into this model a geography that is different from the E&S model. I will provide more detail on these and other differences later in this exposition.

2. The Barter Market

In this paper, I make use of the notion of a barter market in which farmers meet to exchange commodities: i.e., they are both producers and consumers of produce. Put differently, buyers and sellers exchange commodities in barter. While economic actors can meet up to match their needs on a pairwise basis, it is not always clear how an exchange rate (price ratio) gets established between every pair of commodities in a barter economy. Suppose, for example, we have a simple barter market that includes only two farmers. For the sake of argument, assume Farmer A arrives with an endowment of 0.4 units of wheat (commodity 1) and 0.6 units of corn (commodity 2): and that Farmer B has an endowment of 0.60 units of wheat and 0.40 units of corn: q_A1 = 0.40, q_B1 = 0.40, q_A2 = 0.60, and q_B2 = 0.60. In Figure 1, I draw a diagram—called an Edgeworth Box—for this problem. There, point a represents the endowments before any barter. Through point a, I have drawn Person A’s indifference curve (abi_A) before barter. The idea of an Edgeworth Box is to show the indifference curves also for person B; since total endowments are fixed, we can imagine drawing B’s indifference maps from an origin at the upper right hand corner of Figure 1. In that case, B’s indifference curve before barter would be afib^B. The eye-shaped area above abia^A and below afib^B contains all the barter outcomes that would leave each of A and B better off (or at least no worse off) than with without barter.

Where within this “eye” does bartering lead us? There are two answers to this question: Marshallian and Walrasian.

Marshallian answer. Barter leads to an equilibrium in which neither party has an incentive to barter further; that is, the endowments after barter must be such that the indifference curves of A and B are tangential at that point. In Figure 1, I draw the locus of all points that satisfy this; it is labeled bcdef and generally called a Contract Curve. At point b on the Contract Curve, all the gains from barter accrue to Person B. At point f, all the gains accrue to Person A. At any point on the Contract Curve between b and f, both Person A and Person B benefit. In the Marshallian view, where barter ends up along the Contract Curve depends on the bargaining strength of the two participants.

Walrasian answer. Barter leads to the setting of an equilibrium exchange rate between the two commodities. At that exchange rate, the two participants trade commodities until neither party has the incentive to exchange

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1 Economides & Siow (1988) is primarily concerned with the existence and size of financial markets. However, at the outset, that paper describes a simple locational model of trading by farmers that is the focus of this paper. Others who have made use of Economides & Siow (1988) to look at questions of location include Camacho & Persky (1990), Casella (2001), Gehrig (1998), Glazer et al (2003), and Henkel et al (2000).

2 I use barter here in the economic sense of an exchange—a trade of some amount of one good in return solely for an amount of another good with no money involved—that takes place in the context of a market. This is seen here strictly as a matter of business; I exclude here any exchange (e.g., an exchange of gifts) where the motivation is, at least in part, something else.
Further. Therefore, we must reach the point\(^1\) on the offer curve where the Walrasian exchange rate is tangential to the indifference curves of the two persons. In Figure 1, I draw the Walrasian exchange rate as dotted line that passes through the initial endowment (point a) and crosses the Contract Curve at point d where it is tangential to the indifference curves of both persons A and B.\(^2\) In the Walrasian view, barter ends up at point d on the Contract Curve. In moving from a to d, Person A gives up some corn (the same amount purchased by Person B) to purchase some wheat (the amount given up by Person B).

For simplicity of exposition, I adopt the Walrasian view here and find the exchange rate that will leave neither farmer wanting to trade any more.

### 3. Uncertainty and rationality

Economists often characterize markets as varying from thick\(^3\) to thin\(^4\). As a thought experiment—with apologies in advance if the process appears mechanistic or vague—assume an asset market with the following characteristics:

- A large number of potential buyers and a large number of potential sellers. Put differently, the asset is widely desired and widely held.
- The asset is held at zero (storage) cost. However, buyers (sellers) incur transaction costs to acquire (dispose) of the asset.
- Potential buyers and potential sellers each form a statistical population of individuals. Overall, these populations are both of the view that the market price of the asset is not expected to change in the future. However, individuals within either population randomly deviate from this. On any given market day, each potential buyer (seller) has a broad sense of (1) the current price and expectations about the future price of the commodity—expectations that differ randomly from person to person and from day to day—and of (2) the transaction costs associated with their participation in the market.
- At the outset of any given market day, a subset of the potential buyers (call them “market buyers”) and a subset of the potential sellers (call them “market sellers”) engage in the market. By “engage”, I mean they undertake one or more of the following activities: search, gather, and analyze information, make contacts and establish relationships, negotiate price and terms, and acquire/dispose. Why only a subset? In my view, time and effort are required (i.e., transaction costs are incurred) to do these things. To acquire or dispose of a commodity, for example, these would—as noted earlier—include costs related to bank transactions and credit authorization, freight and transfer, storage and inventory, agency and brokerage fees, cost of insurance and other loss risks, installation and removal, warranty and service, and taxes and tariffs. As the day progresses, some market buyers/sellers will transact; others will—on the basis of the information obtained—choose not to transact. For simplicity, I assume that decisions to engage the market in previous days do not affect the transaction costs to be incurred today.
- Other potential buyers and sellers do not participate in the market. I assume here the perceived transaction costs are large enough relative to the gain expected to keep such potential buyers or sellers out of the market that day.
- Except insofar as transaction costs and price expectations in part vary randomly from one person to the next, market buyers and market sellers are no different statistically from other potential buyers and sellers; they can be thought of as just an independently-drawn random sample.
- Market sellers in aggregate have an upward-sloping supply schedule showing the amount they would sell at any given exchange rate.

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\(^1\) Implicit in this description is an assertion that such a point exists and is unique. A determination of the conditions under which this assertion is valid is beyond the scope of this book.

\(^2\) The slope of the Walrasian exchange rate line is the negative of the exchange rate.

\(^3\) A market condition in which there are many buyers and sellers. From a search-theoretic perspective on markets a seller in a thick market does not have to wait long to get a fair price for their good.

\(^4\) A market condition in which there are few buyers and sellers. From a search-theoretic perspective on markets a seller in a thin market typically must wait longer to get a fair price for their good.
• Market buyers in aggregate have a downward-sloping demand schedule showing the amount they would purchase at any given exchange rate.
• Following the Walrasian perspective, the market exchange rate that day settles at a level such that no market seller leaves with product that they would prefer to have sold at that exchange rate and no market buyer leaves without product they would have preferred to have bought at that exchange rate. Put differently, the market clears for market buyers and sellers that day.

For an asset market that can be characterized this way, we would therefore expect to see some variation in market price from day to day even when no one expects price to change over the longer run. We also expect to see the number of market sellers or buyers rise one day and then perhaps fall the next on a random basis. I define the market to be thin when the numbers of market buyers and market sellers are small: thick when the numbers are relatively large. We might expect that the price of the asset in a thick market would be about the same from day to day because of the many participants. In a thin market under similar circumstances, however, we expect the exchange rate of a commodity to vary more from day to day. Put differently, there is more price risk\(^1\) in a thin market than in a thick market; the vendor in a thin market might get less than either potential suppliers or potential suppliers think is the expected price for the asset.

Why engage in a market at all? In general, choosing a market can be seen as a means of reducing or spreading price risk. As a farmer, you do not necessarily need to participate in a weekly farmer’s market. You might have, for example, established relationships with one or more customers who travel to your farm weekly to purchase commodities. Why bother with the inconvenience of shipping to market if you can get a good exchange rate at the farm gate? However, if you don’t attend the market, it is hard to know whether you are getting a good exchange rate; customers too might want to know if they are paying too much. For both farmer and customer, the market provides a means of assessing whether the exchange rate for a given transaction is “fair”. Even a thin market can be helpful here. However, the thicker the market, the less the price risk.

To implement the notion of price risk, we need to think about what it means to be rational under uncertainty.\(^2\) In 1738, Daniel Bernoulli (a Swiss mathematician) developed a theory on the measurement of risk that set the stage for modern approaches.\(^3\) He starts from the notion of an expected value \(E[X]\). For a discrete random variable \((X)\), this is the sum of all possible occurrences of \(X\) each multiplied by the probability, \(p(X)\), of that occurrence: \(E[X] = \sum x(p[X = x])\). However, he then assumes that:

1. the value someone places on an outcome depends not on the expected money gain from a gamble but rather on the utility it yields;
2. the added utility, \(U[X]\), from a given money gain \([X]\) is more for a pauper than for someone who is rich; and
3. the expected value of the gain in utility, \(E[U] = \sum U[x]p[X = x]\), is what motivates individuals in a gamble.

Note the implication here, drawn out by Bernoulli himself (page 29), that no one would therefore rationally gamble in a “fair game”—one in which a dollar gain was as likely as a dollar loss—because the loss has a greater change in utility attached to it than does the gain.

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1 A loss (or increase in cost) arising because of an unforeseen change in market conditions that causes price to change over the short term, price risk is associated with price volatility. In a search-theoretic perspective, sellers hold an asset until the price bid by a potential purchaser exceeds the vendor’s “reservation price”. Here, a distinction can be drawn between price risk and liquidity risk. Liquidity risk is the loss arising because of the delay in obtaining a bid at or above the reservation price. In practice, it is difficult to distinguish between price risk and liquidity risk. The approach in this book is to treat liquidity risk as simply an element of price risk.


3 Bernoulli (1954) is an English translation of that paper.

4 For example, if we toss a fair coin twice and let \(X\) be the number of times a head obtains. \(X\) can take on the values 0, 1, and 2. From the Binomial Theorem, we know that probabilities are \(1/4, 1/2,\) and \(1/4\) respectively. Therefore, \(E(X) = 0(1/4) + (1/2)(1) + (1/4)2 = 1.0.\)
Bernoulli’s assertion (2) above is problematic with respect to the ordinality of utility in two regards. First, in effect, he assumes that individuals have a diminishing marginal utility of income. Why might this be problematic? Diminishing marginal utility of income itself need not be surprising since we commonly assume diminishing marginal utility in commodities consumption. However, in practice, we assume that a utility function is unique up to a monotonic transformation. For example, the utility functions \( f(x, y) = x^b y^c \) where \( 1 < b < 0 \) and \( g(x, y) = ax^b y^c \) where \( b > 0, c > 0, \) and \( b+c < 1, \) calculated at consumption of \( x \) units of wheat and \( y \) units of corn, generate the same rank ordering: i.e., \( g(x, y) \) is a monotonic transformation of \( f(x, y) \). The easiest way to think about diminishing marginal utility of income is that \( b+c < 1, \) but then how would this differ from a monotonic transformation of \( b+c = 1? \) Second, Bernoulli assumes that the utility levels of consumers (the pauper and the rich person) are comparable which again violates the ordinality of utility. Although risk in the context of investment had long been of concern in Economics, it is Von Neumann & Morgenstern’s path-breaking book, *Theory of Games and Economic Behavior*, first published in 1944, that is widely credited with spawning the focus in Economics in general on game theory and in particular on the nature of rational behavior in the presence of uncertainty.\(^1\) That book builds on Bernoulli’s idea that economic actors maximize the expected value of the gain in utility. However, there are problems with expected utility maximization. (1) In practice, how do we find the required probabilities, especially when these may be subjective (Bayesian) in nature? (2) Do individuals have a ‘taste for risk’ or an ‘aversion to risk’ that leads them to prefer one gamble to another even when two gambles have the same expected utility? (3) More generally, why assume that rationality necessarily requires expected utility maximization?

An alternative to analyze rational decision-making under uncertainty is through a mean-variance (or, alternatively, “risk-return”) approach that originated with Markowitz (1952) and Sharpe (1963, 1964). Under this approach, we calculate two measures: (1) the expected utility \( \varepsilon \)—otherwise known as the “mean” or as the “return”—as per Bernoulli, and (2) the variance \( \nu \)—otherwise known as the “risk”—in utility.\(^2\) If two choices have the same return but different risks, the individual is thought to prefer the choice with the lower risk. If two choices have the same risk but different returns, the individual is thought to prefer the alternative with the higher return. If the two choices have different returns and different risks, then we need some way to measure the tradeoff between return and risk. Typically, this is done using what is termed a beta analysis.\(^3\) In this paper, such a risk-return approach is used to characterize rational choice under uncertainty. Here, I distinguish between sub-utility and utility. Sub-utility is the level of happiness that arises from a choice when uncertainty, is, or can be, ignored. Utility is the level of happiness after uncertainty has been taken into account; in a conventional beta analysis, utility is given by (1.1). Beta here is a parameter that measures the aversion of the individual to risk; when \( \beta = 0, \) the individual is indifferent to risk, for larger \( \beta, \) the individual is increasingly averse to choices with substantial risk.\(^4\)

Many advances have been made using a risk-return approach in an area now known broadly as financial engineering. However, the approach is not without its critics. Among these are the following:

- Some economists are not fond of the risk-return approach; they prefer an approach better grounded in neoclassical utility theory. To simplify the investment problem here, imagine an individual choosing between a risky investment with a higher expected return and a risk-free investment with a lower return. In effect, by declining the risky investment, individuals forego an amount (the amount by which the expected return is higher in the risky investment) to guarantee their wealth (the principal invested) at a future date. As such, deciding to invest in the risk-free alternative is like buying insurance and should be analyzable in that way.

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1. In my view, Georgescu-Roegen (1954: page 503) is correct in pointing out that mathematicians and statisticians dating back to Daniel Bernoulli and Gabriel Cramer had worked on similar ideas much before this. Harsanyi (1956) points out the similarities of game theory to earlier work by Zeuthen (1930).

2. That is, \( \varepsilon = \sum U(x)p(X = x) \) and \( \nu = \sum(U(x) - \varepsilon)^2 p(X = x). \)

3. Beta is the increase in mean (return) required to offset a unit increase in variance if two alternatives are to be thought to be equally preferable.

4. This is an approach initially suggested by Markowitz (1976). See also Levy & Markowitz (1979) and Kroll et al (1984).
• The risk-return approach does not directly incorporate an asymmetry (skewedness) of gains and losses as proposed by Bernoulli.

• The risk-return approach—as usually applied in investment analysis—assumes a continuity across investment choices: i.e., the ability to blend investments at differing levels of risk. The location problem that I consider in this paper exhibits a kind of lumpiness that needs to be addressed specifically.

On the other hand, students tell me that they, or their parents, deal with financial advisors who regularly cast investment portfolio choices in terms of risk versus return. Therefore, I find it helpful pedagogically to cast this problem using a risk-return approach.

4. Model A: non-spatial market

Imagine an economy made up of otherwise-identical farmers. Each farmer has the same log-linear utility function for its sub-utility defined over the consumption of two commodities: see (1.2) in Table 1 wherein I summarize equations and notation in Model A. Here, let \( q_1 \) and \( q_2 \) be the amounts of wheat and corn respectively that the farmer consumes and let \( U \) be the level of sub-utility achieved by the farmer. As \( q_1 \) approaches zero in (1.2), so does \( U \); the same is true for \( q_2 \). The model promotes trading by assuming that each farmer is randomly assigned—at harvest time—an initial endowment of one unit of one commodity and none of the other: i.e., \((1, 0)\) or \((0, 1)\). I refer to those with \((1, 0)\) as having a wheat endowment and those with \((0, 1)\) as having a corn endowment. In such circumstances, farmers have an incentive to exchange commodities. If they do not, their utility will be zero. An endowment here is production net of costs; for the moment, I leave unstated exactly how the agricultural commodity is produced except to say that each farmer is efficient and that those who harvest corn (as well as those who harvest wheat) produce an amount of 1 unit of the commodity net of costs including the opportunity cost of land (rent). Further, assume that these initial endowments obtain as though outcomes were random and statistically independent and that for each farmer there is a probability \( \phi \) that he or she will be endowed with corn, and therefore \( 1-\phi \) probability of being endowed with wheat. See (1.3).

In illustrating Model A (and again in Models B and C that follow), I use particular values for \( \nu \) and \( \phi \). I assume \( \nu = 0.3 \), which implies consumers, prefer to consume relatively more wheat than corn. I assume \( \phi = 0.4 \) which means that farmers are more likely to be endowed with wheat than corn. Together, these two values describes a world in which 60% of farmers are endowed with wheat, but where each farmer wants to spend only 30% of his or her endowment on wheat consumption. In that sense, our farmers would be happier if endowments of corn were more commonplace (in other words, if \( 1-\phi \) were closer to \( \nu \)) and less happy otherwise. This further contributes to the imperative to trade. If they do participate in a market, they give up a portion of their initial endowment to get some amount of the other commodity to consume.

Suppose \( N \) farmers constitute a market for this purpose: see (1.4). To simplify the subsequent analysis, I assume farmers decide on their size of market in advance of knowing either their endowment or that of anyone around them (i.e., the endowments of other farmers who might be in the same market). This assumption may seem strange. After all, why not let farmers choose their market later. However, this assumption will make more sense later in this paper when we introduce geography into the model. Having decided on a size of market and having subsequently harvested and determined their endowment, I assume that the farmers then meet in this market, transact as best they can, and get the utility that arises to all farmers with their endowment at the conclusion of the market. As stated above, the endowment for each farmer among these \( N \) is stochastic and independent of the endowment of any other farmer. Then, we can think of the mix of endowments at the marketplace as the outcome of a Bernoulli process\(^2\): i.e., consists of \( N \) independent trials (one for each farmer)

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1 Economides & Siow (1988) also look at the case where the utility function is Constant Elasticity of Substitution.

2 A Bernoulli trial is a statistical experiment which can result in only one of two possible realizations. An experiment consisting of a series of independent Bernoulli trials is called a Bernoulli process.
wherein there is a fixed probability \( \phi \) that a particular outcome—a \((0, 1)\) endowment—occurs.\(^1\) In that case, the market outcomes, as measured by \( N_i \)—the number of occurrences of a wheat farmer—follow a Binomial probability distribution: see (1.5). The number of corn farmers in this market is \( N_2 = N - N_i \), and the fraction that they make of the market is \( k = N_2/N \). Because \( N_i \) is a stochastic variable that is binomially distributed, it has a known expected value, \( E[N_i] \); see (1.6). Because \( N_i \) is a linear function of \( N_i \), it too is a stochastic variable and has a binomial distribution with a calculable expected value.

In a market of size \( N \), we expect (on average) farmers with a wheat endowment and farmers with a corn endowment will each offer in total the amounts of wheat and corn shown in (1.7) in exchange for the other crop. The ratio of expected offers is shown in (1.8).\(^2\) However, the actual exchange rate will differ from the ratio of expected offers because \( k \) can (and often does) differ from \( \phi \). If we have \( N_i \) wheat farmers and \( N_2 \) corn farmers in the market, wheat farmers will offer a total of \((1-\nu)N_i\) units of wheat, corn farmers will offer a total of \(\nu N_2\) units of corn, and on average an equilibrium exchange rate of \((1-\nu)(1-k)/(\nu k)\) will therefore result.

However, we do not always get this average exchange rate: as when farmers arrive at the market to discover to their chagrin that \( k \) is zero or one).\(^3\) Consider a market of just two farmers \((N = 2)\). Remember here that I assume farmers choose a market in advance of knowing their endowment. There are three possible realizations for \( N_i \): \(0, 1, \) or \(2\). When \( \phi = 0.4 \) and \( \nu = 0.3 \), the ratio of expected offers (1.8) is \(3.50\). For each possible outcome of \( N_i \), the corresponding binomial probability is shown in panel (a) of Table 2. In the event, \( N_i = 0 \) or \( N_i = 2 \) here, the market consists entirely of corn endowments or wheat endowments respectively and the utility for each farmer in the market is therefore zero. When \( N = 2 \), there is thus a probability of \(0.52\)\(^4\) that the market participants will have a zero utility: i.e., return home without any of the other commodity. Suppose instead \( N_i = 1 \), which means we have one farmer of each type in the market. The farmer with a wheat endowment consumes 30% of their endowment of wheat, and trades the remaining 70% for corn. The farmer with a corn endowment consumes 70% and trades the remaining 30% for wheat. There are two possible utilities. With a probability \((1-k)\), the farmer will have an endowment of \((1, 0)\), a consumption bundle \((0.3, 0.3)\), and a utility \((U_1)\) of 0.3. With a probability \(k\), the farmer will have an endowment of \((0, 1)\), a consumption bundle \((0.7, 0.7)\) and a utility \((U_2)\) of 0.7. In this situation, the farmer with the corn endowment is better off after trade than the farmer with the wheat endowment: not surprising given that corn is the commodity more strongly preferred. Put differently, the farmer is here guaranteed a utility of 0.3 with a 50% chance of getting 0.7 instead. By calculating \(W[k] = 0.5\)\(^5\), I am simply taking an average of the two possible utilities weighted by the probabilities of the two outcomes. Considered over all the possible realizations of \( k \), the weighted average utility of being in a market of \( N = 2 \), \(U[N]\), from columns \([4]\) and \([11]\) in panel (a) of Table 2, is thus \(0.240\)\(^6\).\(^7\) The variance in this utility, \(V[N]\), is also now calculable; \(V[N] = 0.082\)\(^8\).

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\(^1\) We would not have a Bernoulli process if each individual could wait until harvest time to see his endowment and that of his or her neighbors before deciding in which local market to participate.

\(^2\) Economists usually say in this case that wheat is *numéraire* which means that other goods (corn in this case) are valued in units of wheat.

\(^3\) In an earlier footnote, I raised the question of whether a Walrasian outcome existed and was unique. In the case of a case of a log-linear utility function, the answer intuitively is straightforward. Each farmer maximizes utility by allocating income so that the proportions spent on wheat and corn are \(a\) and \(1-a\) respectively. At the equilibrium exchange rate, \((1-a)/(1-k))/ak\), a Walrasian solution exist; the market clears and farmers of each endowment are as well off as possible. The Walrasian solution is also unique; no other exchange rate clears the market.

\(^4\) \(0.36 + 0.16\).

\(^5\) \(0.5(0.3) + 0.5(0.7)\).

\(^6\) \(0.52(0.00) + 0.48(0.50)\).

\(^7\) This is another place where ordinalists might well cringe. If utility is indeed ordinal, what does it mean to take a linear combination of utilities as we do when we calculated \(U[N]\).

\(^8\) \(0.52(0.00-0.24)^2 + 0.48(0.5(0.30-0.24)^2 + 0.5(0.70-0.24)^2\).
What is the expected exchange rate across the three possible realizations of \( N_i \) from 0 to 2? We cannot calculate an exchange rate when \( k \) is zero or one because no trade happens. In column [4] of panel (a) in Table 2, we see that the probability that \( k \) is zero or one is 0.52. From column [12], we see that at the only other possibility, \( k = 0.50 \), the exchange rate is 2.33 units of wheat per unit of corn. I label this the Conditional Expected Exchange Rate (CEER); that is, the exchange rate we expect on average on the condition that \( k \) is neither 0 nor 1. This is different from the ratio of offers expected, (1.8), which incorporates the amounts offered when \( k = 0 \) and \( k = 1 \). In comparison, the ratio of offers expected is 3.50 when \( \nu = 0.3 \) and \( \phi = 0.4 \) as noted above.

To begin thinking about what might happen if \( N \) were larger than 2, let us do similar calculations for a market of \( N = 8 \) participants. As before, I continue to illustrate using the case where \( \nu = 0.3 \) and \( \phi = 0.4 \). See panel (a) of Table 3. Consider first the case where \( N_i = 3 \). The 3 farmers with a wheat endowment each offer 0.7 units of wheat in exchange for corn. The remaining 5 farmers have a corn endowment; each offers 0.3 units of corn in exchange for wheat. The equilibrium exchange rate here is therefore \( 1.40 \) units of wheat per unit of corn. Each farmer with a wheat endowment therefore consumes 0.3 units of wheat, and 0.5 units of corn for a utility of 0.43. Each farmer with a corn endowment consumes 0.7 units of corn and trades away the remaining 0.3 units in exchange for 0.42 units of wheat to achieve a utility of 0.60. Therefore, the weighted average of utilities, \( W[k] \), is now 0.536. Considered over all the possible realizations of \( k \) as shown in Table 3, the weighted average utility of being in a market of \( N = 8 \), \( U[N] \), from columns [4] and [11] in panel (a) of Table 3, is 0.423 and the variance \( V[N] \) is now 0.087.

What is CEER when \( N = 8 \)? Here we see from column [4] in panel (a) of Table 3 that the probability that \( k \) is zero or one is 0.018, and from column [12] that, for \( k \) in between, the exchange rate varies from 16.33 down to 0.33 which yields CEER = 4.84. This is higher than the 3.50 we noted above for the ratio of expected offers: the opposite of what we had found when \( N = 2 \).

In this model, the farmer chooses the size of market in which to trade. This means the farmer will compare the combination of return, \( U[N] \), and risk, \( V[N] \), with those achievable at other sizes of market. We are now able to compare the case of \( N = 2 \), and \( N = 8 \). When \( N = 8 \), the return is larger (0.423 versus 0.240) but so too is the risk (0.087 versus 0.082) compared to \( N = 2 \). In a risk-return analysis, the farmer would therefore prefer \( N = 8 \) over \( N = 2 \) if beta is sufficiently small and prefer \( N = 2 \) otherwise.

To understand what is happening to CEER, suppose we let \( N \) vary from 2 to 30 and calculate CEER at each \( N \). The resulting estimates of CEER are plotted in Figure 2. The dotted line is the ratio of expected offers (1.8); it is horizontal because this ratio is the same at every \( N \). The solid curve shows CEER as a function of \( N \). CEER is below the ratio of expected offers when \( N = 2 \), rises quickly, and peaks well above the ratio of expected offers at about \( N = 8 \), then begins to fall off asymptotically to the ratio of expected offers as \( N \) becomes large. We are now ready to answer some questions.

- Why is the equilibrium exchange rate low at small \( N \)? This is because (1) the exchange rate is a declining function of \( k \), (2) the equilibrium exchange rate cannot be calculated at \( k = 0 \) or \( k = 1 \), and (3) \( P(k=0) \) is larger than \( P(k=1) \).
- Why is CEER then above the ratio of expected offers for \( N \) sufficiently large (above \( N = 4 \) in Figure 2)? This

1 \( 3(0.7)/(5(0.3)) \).
2 \( 0.7/1.40 \).
3 \( 0.3\times1.40 \).
4 \( (3/8)0.43 + (5/8)0.60 \).
5 In Economics, utility is thought to be an index (or ordering) of preference among choice. As such, utility is an ordinal measure. However when we calculate \( U[N] \), we appear to treat utility as though it were a cardinal measure. For a discussion of the issues raised, see Ellsberg (1954).
6 In this regard, Economides & Siow (1988: page 110) appear to err when they state “Aggregate supply and demand for each commodity are proportional to \( N \), so that the equilibrium price is independent of \( N \).” My sense is that they may be confusing CEER and the ratio of expected offers.
happens because of the asymmetry of an exchange rate. To see this, suppose the quantities of the two commodities offered in the market are identical; the exchange rate here is 1.00. Now consider increasing the quantity of either the numerator or denominator. As we increase the denominator quantity, the exchange rate can drop from 1 to as low as 0. As we increase the numerator quantity, the exchange rate rises from 1 without limit. This asymmetry means that, ignoring the effect of the exclusion of $k = 0$ and $k = 1$ offers, CEER should be systematically higher than the ratio of offers expected in (1.8). However, this bias dissipates as the size of market becomes larger.

Of course, CEER is not the key variable here. To understand how farmers choose markets, we must look at how market size affects utility. In Figure 3, I use a solid line—the risk-return curve—to connect combinations of $U[N]$ and $V[N]$ attainable—for each level of $N$, again from $N = 2$ to $N = 30$. The attainable combination at each $N$ is a black dot on this curve. Further, I have labeled the size of market at selected dots in Figure 3. Here, we see that, as $N$ is increased, the mean increases and the variance starts dropping above $N = 4$. In fact, the locus of points on this curve from $N = 4$ through $N = 30$ suggests that $U[N]$ will continue to increase and $V[N]$ will decrease albeit both ever more slowly as $N$ becomes still larger. In a beta analysis, we assume the farmer is willing to accept a higher risk associated with a higher reward. One such tradeoff curve is shown as a dotted line in Figure 3. We can imagine a family of such dotted lines, all parallel, such that the farmer is happier the higher and to the left the tradeoff curve lies. Given the risk-return curve is negatively sloped above $N = 4$, the farmer would prefer to be in as large a market as possible. This is not surprising. After all, there is no disincentive here to join a larger market, and the larger the market the higher $U[N]$ and the lower $V[N]$.

In Figure 3, I also include the case of autarky ($N = 1$) wherein farmers do not assemble into a market. In autarky under either endowment, the farmer gets a sub-utility of 0 with certainty: i.e., a zero variance. Autarky corresponds to the origin—that is, the intersection of the horizontal and vertical axes—in Figure 3 since $U[N] = 0$ and $V[N] = 0$ there. If we were to draw two risk-return tradeoff lines, each parallel to the dotted line in Figure 3, one drawn through the point where $N = 2$ and the other through the autarky point (the origin) we see that the utility of being in a market of size 2 exceeds the utility of not being in autarky. A similar analysis would lead us to conclude that utility is higher at $N = 3$ and still higher at $N = 4$. Put another way, the slope of the risk-return tradeoff drawn in Figure 3 is sufficiently small (i.e., the risk premium, $\beta$, was low enough) to initiate an agglomeration process.

Does the reverse argument hold true? Is there a $\beta$ sufficiently large that farms might never choose to switch from isolation ($N = 1$) to a market where $N = 2$, or from there to move to markets of $N = 3$ or $N = 4$? Consider the line in Figure 3 joining autarky to $N = 2$. If $\beta$ were sufficiently large to make the risk-return tradeoff line steeper than this, the utility of autarky would appear to be greater than the utility of $N = 2$. Suppose alternatively the risk-return tradeoff line passing through the origin cuts the line segment joining $N = 2$ and $N = 3$. Then the utility of being in a market of $N = 3$ would be greater than the utility of autarky. If so, does that imply that farms would never get to a market size of 3, even though it is advantageous, because no one would first have the incentive to form a market of 2? If so, this would be a disturbing feature of the model because it would imply that planners would have to do what a market could not: i.e., push myopic farmers, unable to see the benefits from further agglomeration, from autarky into a market of $N = 2$ to make it possible for farmers to then form a market of 3 or more.

However, I think the "problem" here, specifically any reluctance to move from autarky to $N = 2$, when $\beta$ is large, is actually indicative of a limitation of risk-return analysis. In autarky, the farmer receives a sub-utility of zero with certainty. When $N = 2$, the farmer receives a sub-utility of zero if both farmers have the same endowment, and a larger sub-utility if they do not. In other words, at the worst, the farmer in a $N = 2$ market does as well as in autarky. In my view, the farmer in an $N = 2$ market is always therefore at least as well off as in autarky and has the possibility of being better off. Even if we assume that the farmer is myopic about the

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1 CEER is not calculable because there is no trade
prospect of more farmers joining the agglomeration and pushing utility even higher, there is an incentive here to form a market of $N = 2$.\footnote{In my view, the "shortcoming" of risk-return analysis here is that the notion of variance loses generality when the number of realizations of a random variable (here, realizations of $k$) is small.}

5. Model B: cooperation in a spatial market

Up until now, we have assumed a non-spatial market. How does space affect this model? Let us now consider how shipping costs would affect the behavior of the bartering farmers here. In Economides & Siow (1988), farms are spread out (at fixed density) along a line left and right of the market point. As I understand the E&S model, this implies that farmers close to the market are better off than farmers who travel a longer distance to get to the market. To establish and maintain farmers in their locations in equilibrium, no farmer should be able to benefit by relocating. There must be some process to ensure that farmers are in equilibrium and do not have an incentive to further relocate. As I understand the E&S model, no such equilibrium process is specified. To correct for this, I first add a cooperative process in this section (Model B) wherein farmers in a market form a club to share shipping costs in the market equally. In the next section (Model C), I introduce an alternative: a noncooperative process wherein farmers nearer the market bid a market rent premium for their plots of land compared to more-remote farms. As it turns out, specifying such processes also helps us better rationalize the idea that farms choose markets in advance of knowing their endowments. I will return to this subject shortly.

In this section, I model farms forming a club or cooperative to share the cost of participating in the market (i.e., shipping costs).\footnote{See McGuire (1972) on economic models of club formation. For other modeling of cooperation in a geographic context, see Jayet (1997) and Soubeyran & Weber (2002).} See Table 4. As used here, a "club" is an association whose purpose is to provide a benefit to its members. In return for paying a membership fee ($f$) each harvest, the club here provides "free" shipping services to the member. A club is like a firm except that it is not intended to make a profit: see (4.6).\footnote{Here I implicitly assume contingent shipping rates. That is, the cost of shipping wheat a kilometer is $s$ units of wheat, and the cost of shipping corn a kilometer is $s$ units of corn. Similarly, the cooperative fee is contingent; it is $f$ units of wheat if the farmer has a wheat endowment, and $f$ units of corn if a corn endowment. There is no adjustment here for the exchange rate between wheat and corn that will obtain in the market. For the storyteller, the advantage of this scheme is that it simplifies decision-making for the farmer who is still in anticipation of the harvest and does not yet know his or her endowment.} Usually, there is an optimal size of club; that is to say, a desired (in this case, most efficient) membership level. As well, implementation of a club also usually involves restrictions on non-members who might otherwise get a "free ride"; that is, benefit from the activities of the club without paying the membership fee. To keep the model simple, I assume that there are no costs to the formation or enforcement of a club other than the cost of shipping. I imagine here that each farmer envisages an optimal size of club and costlessly seeks out peers with a similar view until the optimal number of members has been obtained. If, say, many farmers see an optimal club size of 10 farmers, then additional clubs form will form, each containing 10 farmers. Market formation here is an externality. The action of one farmer in choosing to join a particular market affects the wellbeing of others in a way that is unpriced. The club, as an organizational form, is a mechanism that "captures" (internalizes) this externality or spillover.

Once formed, I assume the club requires its members to locate so as to minimize the total shipping cost that is to be shared. Basically, this means that the farms are tightly packed around the market point; this also helps control the "free rider" problem by keeping nonmembers further away from the market point. In what I characterize as an "accretion process", assume that member farms are therefore required to form concentric rings around the market point. See the three panels in Figure 4 where I map the locations of farms, each farm with the same area, when $N = 6$, $N = 4$, and $N = 2$ respectively. The farmer can be thought to ship from the mid-radius of the farm.\footnote{Mid-radius here is the distance which divides the farm into two equal areas.} The rationale for making each farm annular is that this shape minimizes the cost of shipping...
their endowments to the market. I ignore here other considerations that might "shape" the farm in a geographic sense. In effect, the market is at the centre of the innermost circle: the center of farm 1. Since each farm occupies the same amount of land, the shipping cost associated with the marginal farmer (i.e., the farmer furthest from the market) and the fee (average cost) paid by each farmer in a cooperative increases at a decreasing rate with the number of farmers in the market. See Figure 5. As I have already assumed land is plentiful, I don't need to worry about the possibility of clubs with overlapping market areas; a club would simply move to an unoccupied area so that it can achieve the same low total shipping cost as any other club of the same size.

In this model, farmers may differ from one another in the following ways: (1) a randomly-determined endowment not known in advance of club formation; (2) a given tradeoff between risk and return; and (3) a location vis-à-vis the market that the farmer can choose. Otherwise, I assume farmers are identical: same preferences for wheat and corn; same fee to join a given cooperative. Therefore, a club will be formed by farmers with similar tradeoffs between risk and return: i.e., similar βs. Once in a club, the farmer is indifferent as to location because the club pays the marginal cost of shipping from that site to the market. Assume each farmer uses a fixed amount of land, 1/g, in agricultural production. Assume that land is not used for any other purpose (we ignore here any need for land for transportation, for a market site, for housing, or for the production of any other commodity). On a two-dimensional plane, we can therefore imagine farmers spread at a fixed density: g farmers per unit area. If N is the number of farmers participating in a market, then the outer radius of the market (i.e., the marginal farm) is X = (N/(rg))^{0.5}. Assume the cooperative incurs a constant unit shipping rate for each of its member farmers. Therefore, for the marginal farmer at distance x from the market, the cost of shipping to the market incurred by the cooperative is sx where s is the unit shipping rate per kilometer shipped (assumed to be the same for both commodities). In the empirical modeling that follows, suppose g = 30 and s = 0.40. The first farm (area of 1/g = 0.033) stretches from an inner radius of 0 to an outer radius 0.1030; its mid-radius is 0.0728 and the shipping cost incurred by the cooperative for it is 0.0291. The second farm (also an area of 1/g = 0.033) stretches from radius 0.1030 to radius 0.1457; its mid-radius is 0.1262 and the marginal shipping cost for the cooperative is therefore 0.0505. See (4.1) and (4.2).

I treat the membership fee (measured in units of the endowment and paid at the time of the harvest) as a constant per farmer. For the cooperative, this fee is simply cost recovery: i.e., total shipping cost incurred by the cooperative is split equally among the N farmers in the cooperative: see (4.6).

A feature of the log-linear utility function is that consumption has a linear expansion path. Even though the farmer's income available for consumption of wheat and corn is now net of the membership fee, consumption changes proportionally with income; the marginal farmer still spends the same proportion (ν) of its net income on wheat and the remainder on corn.

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1 For instance, while a ring might be the most efficient shape for getting the agricultural commodity to market, it may be inefficient for the daily chores of the farmer throughout the growing season. Thünen (1966, Chapters 11 and 13) discusses aspects of this problem.

2 This might be because each farmer has a Leontief technology that requires all inputs be in fixed proportion; however, the model to this stage is silent on other inputs to production.

3 Economides & Slow (1988) model the case where farms are spread out along a line in one-dimensional space.

4 (0.40)(0.0728).

5 (0.40)(0.1262).

6 Since each farmer has an endowment of either (1, 0) or (0, 1), if I assume that each farmer carries their entire endowment to the market to trade (not unreasonable given that the farmer does not know the exchange rate that might be established, the shipping cost associated with each farmer is fixed whether measured per unit shipped or per farmer.

7 As used here, a condition of the utility function wherein, if as income is increased by a fixed proportion holding prices of commodities constant, the rational consumer purchases the same proportion more of each good. Put differently, each good has an income elasticity of +1.0. Such a utility function is also said to exhibit homotheticity.
Here in Model B, I assume farmers cooperate by sharing equally the total shipping costs of all farmers in the market. If market size (N) is just 2 farms, the co-op fee (f) borne by each farmer would be 0.0398. Suppose \( N_1 = 1 \). One farmer has an initial endowment (income net of production cost and land rent) of \((1, 0)\). Since \( v = 0.3 \), this person prefers to consume 30% of their endowment net of cooperative fee in wheat itself, and trade the remaining 70% away for corn. See panel (b) of Table 2. The other farmer has an initial endowment of \((0, 1)\) of which he or she prefers to consume 70% (again net of cooperative fee) and trade away the remaining 30% for wheat. See (4.3) through (4.5). Since each farmer pays a cooperative fee equal to the average shipping cost, that exchange ratio between wheat and corn in this market is \( 2.33 \). Where market size is just 2 farms, the co-op fee (f) borne by each farmer would be 0.0398, down from 0.240 in Model A. Further, the variance in utility, \( V[N] \), is also now calculable; \( V[N] = 0.075 \) down from 0.082 in Model A. To conclude, in the case of \( N = 2 \), the introduction of shipping cost reduces both return and risk compared to Model A.

Now let us do similar calculations for a market of \( N = 8 \) participants: See panel (b) of Table 3. CEER remains the same as in Model A. However, compared to Model A in panel (a), we find—as when \( N = 2 \)—that \( U_1, U_2 \), and \( W[k] \) drop for any \( 0 < k < 1 \). As a result, \( U[N] = 0.390 \) is smaller than for Model A. \( V[N] \) too is smaller than in Model A. As in \( N = 2 \), return and risk are both smaller once we take shipping cost into account.

We can then compare this combination of return and risk with those achievable at other sizes of market. I repeated the same process of calculating \( U[N] \) and \( V[N] \), as described above, for all market sizes from \( N = 2 \) to \( N = 30 \) in the spatial case. See Figure 6. There, I show the risk-return curve as a grey line joining achievable combination at each market size in Model A (reproduced from Figure 3) and as a black line joining white dots (labeled Model B). I have also reproduced the risk-return tradeoff from Figure 3 For the risk-return curves in Models A and B, I have labeled selected market sizes from 2 to 30. Here, we see the effects of introducing shipping cost.

1. When \( N = 1 \), there is no effect since each farmer is in autarky.
2. For \( N \geq 2 \), shipping cost pulls the risk-return curve downward and to the left: i.e., lower return and lower risk once shipping cost is incorporated.
3. For \( N \geq 2 \) but small, the introduction of shipping cost reduces both risk and return by a relatively small amount.
4. For \( N \geq 2 \) and large. Previously in Model A, a larger market meant unequivocally lower risk and higher return. In Model B however, this is no longer true. When market size is large, the reductions in both risk and return are relatively large. The geography of a larger market area may mean that the cost of shipping eventually reaches a magnitude where it undermines the advantage of further increasing market size.

In the hypothetical example shown in Figure 6, in the presence of shipping costs, the marginal farmer with a beta associated with the risk-return tradeoff line displayed would find it best to participate in a cooperative of 14 farmers. In this way, shipping costs help explain why market size is not unlimited. Model A suggests farmers

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1. \( (0.0505+0.0291)/2 \).
2. \( 0.70(1-0.398)/(0.30(1-0.0398)) \).
3. \( 0.5(0.29) + 0.5(0.67) \).
4. \( 0.52(0.00) + 0.48(0.480) \).
5. \( 0.52(0.00-0.23)^2 + 0.48(0.5(0.29-0.23)^2 + 0.5(0.67-0.23)^2) \).

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6. In Model B, the shape of the risk return curve is sensitive to the unit shipping rate, \( s \). As \( s \) approaches zero, the risk return curve approaches that for Model A in Figure 9.6. On the other hand, as \( s \) is made larger, the risk-return curve for Model B is pulled even further back and down at larger \( N \).
have an unlimited appetite for participating in large markets. However, shipping costs in the form of membership fees curb this appetite. Size of market reflects the offsetting influences of return and risk on the one hand and average shipping costs on the other. If farmers have a higher beta—that is, a greater aversion to risk, the risk-return tradeoff line in Figure 6 would be steeper and farmers would then opt for a cooperative with a larger N. See (4.7) and (4.8). In other words, farmers would then prefer a lower level of risk even though that might involve a substantially lower return.

Earlier, I had said that farmers with a similar $\beta$ would form a club. As is evident from Figure 6, that is not always the case. If $s$ is sufficiently small, every farmer would prefer an infinitely large market and we might therefore find a great mix of $s$ among farmers in any club so formed. It is more correct to say that, if $s$ is sufficiently large, the risk-return curve under Model B will consist of a set of distinct market sizes each of which will make that size best for farmers within an interval of $\beta$. In that sense, Model B is “lumpy”. I draw the risk-return curve as a polyline in both Figure 3 and Figure 6; however, this is just for ease of exposition. In fact, the risk = return curve is just a set of combinations of $U[N]$ and $V[N]$: one for each integer size of market: the line segments joining them have no particular meaning. When, in a beta analysis, we draw a risk-return tradeoff as an upward sloped line, we are asking simply which combination of $U[N]$ and $V[N]$ on the risk-return curve allows the farmer to reach the highest risk-return tradeoff. In the example shown in Figure 7 for instance, farmers whose $\beta$ is above 0.0873 would choose at least $N=12$ (since that is the slope of risk and return joining $N=12$ to $N=11$); farmers whose $\beta$ is below 0.3168 would choose no more than $N=12$ (since that is the slope of risk and return joining $N=12$ to $N=13$). In Figure 8, I show a step function from which we can read, for any given $\beta$, the appropriate size of market in this example; for instance, at $\beta=1.4$, the farmer chooses a market of 20 in Model B. Others prefer to tell economic stories without such step functions (lumpiness); they would like something akin to Model B but wherein size of market, $N$, was a continuous variable that could be analyzed more easily using calculus or other methods that rely on continuity. However, I like Model B from a pedagogical perspective because it clarifies just how a farm might decide in practice whether to join a given cooperative.

What about comparative statics in this model?

$g$
In Model B, $g$ appears to be measuring something similar to $s$. As $s$ and thereby $f$ is increased, the loss associated with the cost of shipping increases. As $g$ is decreased, the density of farms declines and the marginal farmer has to travel further to participate in a market of size $N$ and thereby $f$ is increased. Put differently, the loss associated with shipping (and hence the cooperative fee) increases when $g$ is decreased.

$s$
In the non-spatial world of Model A, a beta analysis of risk and return would lead farmers to congregate in a single global market since the risk return curve “bends up”. If $s$ is close enough to zero, that may also happen in Model B. In Model B, $s$ must be sufficiently large to cause the risk-return curve to have a positive slope (“bend back down”) for sufficiently large $N$ before we will observe farmers choosing to form a club: i.e., a local (smaller) market. Put differently, if we decrease $s$ and thereby $f$, the farm has an incentive to travel further to benefit from a larger market, and for a sufficiently small $s$ and thereby $f$, there will be a single global market.

$\beta$
If $\beta$ is increased, farmers become more risk averse. They are willing to spend more to join a larger cooperative because they attach more importance to reducing risk. Provided $s$ is sufficiently large to make the risk return curve bend back down enough so that a risk-return tradeoff line can be tangential to it, an increase in $b$ causes farmers to prefer a larger market.

$\nu$
If $\nu$ is increased, each farmer prefers more wheat relative to corn. This causes CEER to fall since farmers now see wheat as more valuable. However, it has no effect on the efficient size of market, $N$.

$\phi$
If $\phi$ is increased, a farmer is more likely to have a corn endowment. Effects on the model depend on the size of $\phi$ relative to $1-\nu$. If $\phi$ is smaller than $1-\nu$, an increase in $\phi$ brings the ratio of corn to wheat endowments closer to what consumers would prefer. If $\phi$ is larger than $1-\nu$, an increase in $\phi$ pushes the
ratio of corn to wheat endowments higher than what consumers would prefer. Size of market is unaffected.

Why do farmers here seek to cooperate? What is driving them here is the risk associated with the randomness of the harvest: i.e., their endowment. To form a club, as is done in Model B, is just one possible response to that risk. What might be some other possibilities?

- **Crop insurance.** If farmers need to consume both corn and wheat, and plant both crops, it would be reasonable for them to purchase insurance against crop failure. Model B does not include this possibility, so the alternative is to band together with other farmers hoping that there will be sufficient amounts of both wheat and corn at the market to enable a good trade. Presumably, crop insurance is structured to give farmers (every farmer in this case) an amount of the commodity they fail to harvest. Presumably the insurer collects a premium from each farmer, awards every farmer an insurance benefit, and still manages to keep something for itself (e.g., profit and wages for the insurer and its workers). Under Model B, each farm pays its share of shipping costs (which is like an insurance premium). Each farm also receives back some amount of the commodity that they did not harvest (except when k is zero or one). In that sense, the farmers in Model B have greater risks than they would with crop insurance (since they could still arrive at a market where k was near 0 or near 1, but less risk than they would have in the absence of both a commodity market and crop insurance.

- **Arbitrage.** Model B assumes there is no way to purchase commodities other than by traveling to a market. As envisaged here, there would potentially be a large number of local markets each with its own exchange rate between wheat and corn. Where exchange rates differ between two local markets by more than the unit shipping cost between markets, presumably arbitrageurs would have an incentive to enter. If the farmer knows that there are competitive traders lurking in every market looking for such opportunities, they will have an incentive to choose a smaller market and save on the cost of shipping to the extent that they know traders will keep the local exchange rate from getting too unfavorable.

At the outset of this paper, I suggested that the organization of the firm was itself endogenous to locational competition. If we think of farmers as firms, and the cooperative as an extension of the activities of a firm, Model B offers a new insight in this matter. With the creation of the cooperative, each farmer is agreeing to be bound by conditions and restrictions of that group. In effect, the firm’s pursuit of its own well-being is now spread across two establishments: one at the farm site and one at the level of the cooperative (market). In effect, the cooperative helps each farmer achieve a better market outcome than they might otherwise.

In the Marshallian sense, can Model B be thought of as a localization economy? I think the answer to that is yes. In Model B, the cooperative spreads (shipping) cost among participants and clusters farms. The process is endogenous; farmers choose to participate in a cluster (or not) not by fiat but on the basis of their own assessment of costs and benefits. Here although farmers close to the market point pay a greater co-op fee than they recoup in shipping cost reimbursed, there is no incentive to leave the cooperative. Even though they are cross-subsidizing the more remote farmers, they know that the cooperative has pushed those farmers into concentric rings that are the most efficient and that they could not get the efficiencies of a market of this size without the cooperative.

Let me expand here on the idea of a club here. As presented, the cooperative internalizes the externality of market formation by sharing shipping cost and ensuring efficient location. In a more general sense, local government generally can be thought to (1) address local externalities through infrastructure investment, service provision, and activity regulation and to (2) redistribute costs among municipal revenue sources such as property tax, local sales tax, and other sources in addition to user fees. In this sense, a club can be thought of as a metaphor for local government. Put differently, cooperatives serve to separate farmers by risk category; local governments can perform a similar function.

### 6. Model C: competition for land in a spatial market

Now, consider a model in which there is no cooperation among farmers. Instead, assume farms choose a market and bid for a location in proximity to that market in the knowledge that each must pay their own
shipping cost to get their endowment to the market. See (5.1) and (5.2) in Table 5. As in Model B, I assume that unit shipping cost is contingent: i.e., a fixed proportion of the farmer’s endowment that is independent of the exchange rate that gets established for wheat and corn.

In a competitive process, I imagine that market rent for land rises above the opportunity cost of land (paid by farmers in Models A and B) insofar as “scarcity rents” arise around market points. In this model, landlords are again absentee in the sense that rent payments to them disappear and do not affect the exchange rate in any local market. I also assume that all rent payments are contingent; the farmer’s rent is a fixed proportion of his or her endowment regardless of the exchange rate that gets established in any local market.

In Model C, I retain the assumption that farms each form a concentric ring around the market point. After all, shipping cost now gives the farm an incentive to want to be near the market point. Between any pair of farm sites (rings), competitive bidding for land means the difference in rent in equilibrium must be just enough to make the farmer indifferent between the two sites. Rent for the farmer at the boundary of the market area is zero; see (5.4). Further, since I assume the amount of land used by a farmer is fixed, the equilibrium difference in rent (per harvest) between the two sites must exactly offset any savings in the cost of shipping. That rent plus shipping cost must therefore total to a constant is the so-called Wingo condition.1 Rent for the farmer in ring i is given by (5.5). Once scarcity rents reach this level, there is no incentive for a farmer to prefer any one ring to another within a given market. However, there will still be differences among markets of different size. The implication of (5.5) is that, the larger is N, the higher will be the rent in ring 1.

Given a market of size N, there are important differences between Models C and B. In Model C, the farmer at ring N loses from his or her endowment a shipping cost of \( sv((N-0.5)/NG) \), zero rent, and (needless to say here in Model C) there is no cooperative fee. In Model B, the same farmer incurs zero shipping cost, zero scarcity rent, and a cooperative fee of \( (1/N)Σsv((i-0.5)/MG) \). At ring N, the loss to endowment is smaller in Model B than in Model C. At any location closer to the market, the loss to endowment for the farmer in Model B stays the same; so too does the loss to endowment for the farmer in Model C since rent there increases to offset any savings in shipping cost. Therefore, the farmer in Model B is better off than the farmer in Model C by the same amount regardless of location in a market of a given size. Why is this? It is because absentee landlords are collecting scarcity rents in Model C (losses to farmers) that do not appear in Model B.

To begin an interpretation of Model C, consider panel (c) of Table 2 wherein \( N = 2 \). Suppose \( N_1 = 1 \). The wheat farmer has an initial endowment (income) of \((1, 0)\). Continuing the assumption that \( v = 0.3 \), this person prefers to consume 30% of their endowment (after rent and shipping cost) of wheat, and trade the remaining 70% away for corn. The corn farmer has an initial endowment of \((0, 1)\) of which he or she prefers to consume 70% (again after rent and shipping cost) and trade away the remaining 30% for wheat. Since each farmer, regardless of location, pays the same total of rent and shipping cost, that exchange ratio between wheat and corn in this market is still 2.33 units of wheat per unit of corn just as in Models A and B. There are two possible utilities when \( k = 0.5 \): a \((1, 0)\) endowment that yields a consumption bundle \((0.28, 0.28)\) and a utility \( U_1 \) of 0.28 (compared to 0.29 and 0.30 in Models B and A respectively) with a probability \( k \) of 0.5 or a consumption bundle \((0.66, 0.66)\) and a utility \( U_2 \) of 0.66 (compared with 0.67 or 0.70 in Models B or A respectively) with a probability \( (1-k) \) of 0.5. Then, \( W[k] = 0.475^2 \) (compared to 0.48 or 0.50 in Models B or A respectively). The introduction of shipping cost here leaves the exchange rate unchanged but reduces the sub-utility in every outcome; not surprising given that rent and shipping cost reduces the amount of wheat and corn available for consumption. As a result, \( U[N] \), from columns [4] and [11] in panel (b) of Table 2 is thus 0.2287, down from 0.230 or 0.240 in Models B or A respectively. Further, the variance in this utility, \( V[N] \), is also now calculable; \( V[N] = \)

---

1 Wingo (1961) originated the idea that land rent offsets transportation cost savings. Later, Alonso (1964) argued that the relationship between land rent and transportation cost savings was also affected by the elasticity of substitution between land and other commodities. Since I have here assumed that the amount of land used by each farmer is fixed, I don’t have to take elasticity of substitution into account.

2 \( 0.5(0.27) + 0.5(0.66) \).

3 \( 0.52(0.00) + 0.48(0.475) \).
0.074\(^1\) down from 0.075 or 0.082 in Models B or A respectively. To conclude, in the case of \(N = 2\), the introduction of rent and shipping costs reduces both return and risk compared to Models B and A.

Now let us do similar calculations for a spatial market of \(N = 8\) participants: See panel (c) of Table 3. CEER remains the same as in Models B and A. However, compared to Models A and B we find—as when \(N = 2\)—that \(U_1\), \(U_2\) and \(W[k]\) drop for any \(0 < k < 1\). As a result, \(U[N] = 0.375\) is smaller than it was in either Models B or A. \(V[N]\) too is smaller than in either Models B or A. As in \(N = 2\), return and risk are both smaller once we take rent and shipping cost into account. A further implication is that, in a risk-return analysis, the farmer would prefer \(N = 8\) over \(N = 2\) in Model C.

We can then compare this combination of return and risk with those achievable at other sizes of market. I repeated the same process of calculating \(U[N]\) and \(V[N]\), as described above, for all market sizes from \(N = 2\) to \(N = 30\) in Model C. See Figure 6. There, I show the risk-return curves for Model C in addition to Models B and A. Here, we see the effects of introducing rent and shipping cost are to cause the risk-return curve to bend back down (have a positive slope when \(N\) is sufficiently large) even more than in Model B. In the example shown in Figure 6, in the presence of rent and shipping costs, the marginal farmer with a beta associated with the risk-return tradeoff line displayed would find it best to participate in a cooperative of just 10 farmers. In this way, rent and shipping costs help explain why market size is not unlimited. Model A suggests farmers have an unlimited appetite for participating in large markets. Model B shows that shipping costs curb this appetite. In the presence of land rent, the efficient size of market shrinks even further in Model C where competition rather than cooperation is seen to underlie a firm’s behavior.

In the example of Model C illustrated in Figure 6, firms with the beta illustrated by the slope of the risk-return tradeoff choose \(N=10\), compared to \(N=14\) in Model B. In Figure 8, I show that the best size of market is consistently lower in Model C than in Model B at every level of \(\beta\). The reason is simple; the farmer in Model C finds it more costly to participate in a market of a given size (because they pay the full shipping cost from the edge of the market area) compared to the farmer in Model B (who pays only the average shipping cost), and therefore chooses a smaller size of market at any given level of risk aversion. In effect, the mechanism of land rent serves to separate farmers by risk category just as do cooperatives.

It is typically argued that competitive markets are efficient. Is that always the case here? The answer—perhaps surprisingly—is no. Specifically, a farm with a beta corresponding to the risk-return tradeoff shown in Figure 6 would be better off in Model B than in Model C. Put differently, the outcomes in the cooperative scheme in Model B appear to be more efficient than in the competitive scheme in Model C. Why is this? Two related answers come to mind. First, Model B may only appear to be more efficient because it ignores the costs of organizing cooperatives. If such costs were substantial, they could drag the risk return curve for Model B below that for Model C; making the competitive solution the more efficient. Second, under the cooperative scheme, farms organize themselves into a larger unit to reap the advantages brought about by the reduction of risk. In this sense, the Coasian view would be that the firm is merely internalizing something (here, a reduced risk) that might otherwise be unavailable or costly to provide in a competitive market. Put differently, belonging to a collective is integral to firm organization: allowing the firm to be efficient.

Models B and C say something new about why individuals participate in a local market. In this paper, the farmer can be thought to make a choice between a smaller market and a larger market in terms of the of risk and return associated with each.

In Model C, land rents vary by location within the regional economy. We can think here of a farm at the edge of its market area (i.e., the \(N\)th distant farmer in a market of \(N\) farms) as a marginal farm. Compared to the marginal farm, the endowments of farms that are closer to the market (i.e., farms 1 through \(N-1\)) now can be seen to include an “excess utility” that arises because of a savings in shipping cost. In Model C, the excess utility gets bid away as land rents so that the first \(N-1\) farmers end up only as well off as the marginal farmer in their market. Put differently, these land rents do not arise because of something that landlords have done to

\[^{1}\ 0.52(0.00-0.228)^2 + 0.48(0.5(0.28-0.228)^2 + 0.5(0.66-0.228)^2).\]
make their properties individually more attractive to tenants. Instead, they arise because of the clustering of N farms. To farms, they are a loss in consumption (a leakage to the farm economy) that happens because farms compete in the land market. The model is silent on how farms produce their endowments. However, we could readily imagine the farm as an enterprise that uses capital, labor, and land to produce its wheat or corn. If so, Model C hints at a fundamental underlying relationship among factor payments to these inputs. In Model C, the gain in utility from participating in a larger market is divvied up between (1) implicit returns to labor and capital in the farm enterprise and (2) the return to land.

What about the comparative statics of Model C? Once we take into account that the cooperative fee in Model B is now replaced by shipping cost and land rent, the comparative statics are much the same as in Model B. Why is that? In part, it is because Models B and C have the same parameters: $\nu$, $\beta$, $\phi$, $g$, and $s$. In Model B, the farmer pays a fixed cost (the cooperative fee) regardless of location that increases with the size of the market. In Model C, the farmer also pays a fixed total cost (for shipping plus land rent) that increases with the size of the market.

**g** In Model C, as $g$ is decreased, the density of farms declines and the marginal farmer has to travel further to participate in a market of size $N$ and thereby shipping cost is increased. Put differently, the loss associated with shipping and/or rent increases when $g$ is decreased.

**s** If $s$ is near zero, farmers in Model C form a single global market because the risk return curve “bends up”. If $s$ is large enough to cause the risk-return curve to have a positive slope (“bend back down”) for sufficiently large $N$, we observe farmers choosing to form a local (smaller) club. Put differently, if we decrease $s$, the farm has an incentive to travel further to benefit from a larger market, and for a sufficiently small $s$, there will be a single global market.

**$\beta$** If $\beta$ is increased, farmers become more risk averse. They are willing to spend more to join a larger cooperative because they attach more importance to reducing risk. Provided $s$ is sufficiently large to make the risk return curve bend back down enough so that a risk-return tradeoff line can be tangential to it, an increase in $\beta$ causes farmers to prefer a larger market.

**$\nu$** If $\nu$ is increased, each farmer prefers more wheat relative to corn. This causes CEER to fall since farmers now see wheat as more valuable. However, it has no effect on the efficient size of market, $N$.

**$\phi$** If $\phi$ is increased, a farmer is more likely to have a corn endowment. Effects on the model depend on the size of $\phi$ relative to $1-\nu$. If $\phi$ is smaller than $1-\nu$, an increase in $\phi$ brings the ratio of corn to wheat endowments closer to what consumers would prefer. If $\phi$ is larger than $1-\nu$, an increase in $\phi$ pushes the ratio of corn to wheat endowments higher than what consumers would prefer. Size of market is unaffected.

Before leaving this section, let me add one thought about landlords. As envisaged here, there will be a mix of markets across the landscape; some small and some large. Corresponding to these will be a mix of landlords; some will receive a high scarcity rent for their site because it happens to be near the center of a large market; others will receive only a rent equal to the opportunity cost of the land: i.e., a zero scarcity rent. If landlords themselves were competitive, why wouldn’t a landlord with a zero scarcity rent seek to entice farmers to relocate more advantageously? In the absence of collusion and given enough landlords (who each own only a small amount of land), there is no mechanism by which this could be achieved. At the same time, the potential for a higher land rent might in practice create an incentive for landlords to collude locally to ensure that they attract and hold the scarcity rents associated with a larger market.

**7. Final comments**

In this paper, the principal model has been C. I included Models A and B to help readers better understand aspects of Model C. The models differ in that (i) Model A assumes shipping costs are zero, (ii) Model B assumes that farms address differences in shipping costs with location by forming cooperatives, and (iii) Model C assumes that farms address differences in shipping costs by bidding up the price (rent) for land at advantageous locations.
I end this paper with six sets of thoughts.

First, I introduced the idea of the importance of transaction cost in location theory. I defined there the concept of an “effective price” which includes the price paid plus the unit transaction costs incurred by the purchaser related to search and information gathering, negotiation, and acquisition, inclusive of normal profit. In Model B here, the cooperative fee can be thought of as a transaction cost. In Model C, the combination of shipping cost and rent can similarly be thought to be the transaction cost. In both models, the farmer is free to choose a level of transaction cost (i.e., a size of market or location). At the same time, there is nothing to guarantee that the farmer will always get a better price in a larger market. In these respects, choosing a larger market is like searching. The farmer in a larger market is getting to see a wider cross-section of market participants. This is like (but not the same as) a consumer gathering information from different suppliers before deciding whether and from whom to purchase. The reader might therefore be tempted to apply this model to help in understanding search behavior. However, there are important differences between market formation as modeled here and conventional thought about search behavior. Principal among these in my mind is the idea that search is often seen as a sequential process. In my view, the consumer gathers information about another supplier or commodity, then makes a decision about whether to continue searching, to purchase, or give up on the idea of purchasing such a commodity. In contrast, the farmer in Models B and C is simply making a gamble among sizes of market; there is no sequential process at work here.

Second, the farmer in this paper is always assumed to use the same amount of land in production. However, in Model C, land becomes relatively more costly to rent, the closer the farm is to the market. Presumably, when an input like land becomes more costly, we might expect the farmer to change the way in which he or she produces agricultural commodities. One simple way to do this is to switch between production that is less labor-intensive (when land rents are low) and production that is more labor-intensive (when land rents are high). To do this, our model of the farm would have to include a production function that enables a substitution between land and labor. The models in this paper assume a fixed amount of land per farm and are silent on labor input. From my perspective, the easiest way to think about the farmer in this paper is that he or she constitutes one unit of labor and that the farm has a Leontief production technology that requires exactly 1 unit of labor and 1/g units of land to produce stochastically an endowment of either (1, 0) or (0, 1)

Third, we began Model A with the apparently-innocent assumption that each farm had an independently-drawn random endowment of 1 unit of either wheat or corn. Implicit in this assumption is the idea that the scale of a farm is somehow fixed. Imagine how much different the model might look if we started with the assumption that the farm had an independently-drawn random endowment of 1 unit of either wheat or corn for each unit of land farmed and was free to hire labor and rent land as needed. In that case, a farm could largely (if not entirely) eliminate the need to travel to a market by being of sufficient size to render insignificant the risk of being without suitable quantities of the two goods. In effect, the farm uses its own size to “self-insure” rather than rely on what is essentially an insurance function performed by the market. The gain to the farmer here is the higher level of utility now possible in the absence of scarcity rents.

Fourth, I am able to solve the models in this paper because I took a Walrasian perspective on exchange rates: i.e., I found the exchange rate that leaves neither kind of farmer feeling that there is still some amount of one commodity that they would prefer to trade for the other. However, as noted at the outset of the paper, the Walrasian solution is just one point on a Marshallian Contract Curve. At one level, one might argue that this indeterminacy of outcome makes the Marshallian approach less useful than a Walrasian approach. One might also argue that, as the number of market participants becomes large, the Marshallian solution converges on the Walrasian solution. Nonetheless, it seems to me unreasonable to believe that farmers will entirely ignore the prospect that, in a small market, their bargaining power (however derived) may leave them at an effective exchange rate different from that envisaged by Walras. The models in the paper do not consider this.

Fifth, what about Walrasian equilibrium across markets? In Model A, there is only one market: a global market for the exchange of wheat for corn. Walrasian equilibrium is irrelevant here. In Model B, unit shipping costs drive farms to organize themselves into local markets for the exchange of wheat for corn. Because the exchange rate realized in each local market is subject to random variation, we can say only that this particular
The process of choosing local markets leaves individual farms best off viewed over a longer term of repeated trials. The same is true of Model C with a twist; in C, there is a market for land as well as local markets for the exchange of wheat for soap.

Sixth, this paper gives us some hints about the regional economy. However, much remains to be done to flesh this out. We need to know more about the process by which endowments of wheat and corn are generated. It would be helpful for example to know about the use of labor, capital, and land inputs in the production process. That would make it possible for us to better understand the regional economy and the impacts of changes in givens on regional well being.
Table 1  Model A: Farmers and the market in a non-spatial economy

Utility of farmer
\[ U = \beta V \]  (1.1)

Sub-utility of farmer
\[ U = q_1^{1-\nu}q_2^\nu \]  (1.2)

Initial endowment of farmer
\[ P[1, 0] = 1 - \phi \quad \text{and} \quad P[0, 1] = \phi \]  (1.3)

Mix of farmers in market
\[ N = N_1 + N_2 \]  (1.4)

Probability of getting \( N_1 \) wheat farmers in market
\[ P[N_1] = C_{N_1}^N(1-\phi)^{N_1}\phi^{N-N_1} \quad \text{where} \quad C_{N_1}^N = N!/(N_1!(N-N_1)!) \]  (1.5)

Expected number of wheat and corn farmers in market
\[ E[N_1] = (1-\phi)N \quad E[N_2] = \phi N \]  (1.6)

Wheat and corn expected to be offered in market
\[ E[Q_1] = (1-\nu)(1-\phi)N \quad E[Q_2] = \nu\phi N \]  (1.7)

Ratio of expected offers (wheat per unit of corn)
\[ E[Q_1]/E[Q_2] = (1-\nu)(1-\phi)/\nu\phi \]  (1.8)

Notation

Givens (parameter or exogenous)

- \( N \): Number of farmers in market
- \( \beta \): Risk aversion parameter
- \( \nu \): Exponent of wheat in utility function. Relative preference for wheat
- \( \phi \): Probability farmer has \((0, 1)\) endowment

Outcomes (endogenous)

- \( N_1 \): Number of wheat farmers in market
- \( N_2 \): Number of corn farmers in market
- \( Q_1 \): Aggregate quantity of wheat offered in market
- \( q_1 \): Quantity of wheat consumed by farmer
- \( Q_2 \): Aggregate quantity of corn offered in market
- \( q_2 \): Quantity of corn consumed by farmer
- \( U \): Sub-utility of farmer
- \( V \): Variance

Note

- \( C \): Combination symbol: \( C_a^b = b!/(a!(b-a)!) \)
- \( P \): Probability
Table 2  Possible realizations in market of size 2 wherein $\nu = 0.3$ and $\phi = 0.4$

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Expected value 0.240 2.33
Variance 0.082

(b) Model B (Spatial: $g = 30$, $r = 0$, $s = 0.40$)

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Expected value 0.230 2.33
Variance 0.075

(c) Model C (Spatial: $g = 30$, $s = 0.40$)

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Expected value 0.228 2.33
Variance 0.074

Note Calculations by author. See also Table 1. In panel (b), mean unit shipping cost is 0.0689.

**Indicates no one of that type present. See also Table 1.**

**Ex** Exchange rate: units of wheat per unit of corn

**Endogenous**

- **U[N]** Utility of being in market of size $N$
- **V[N]** Variance in utility in market of size $N$
- **W** Average utility weighted by number of farmers of each type.
### Table 3  
Possible realizations in market of size 8 wherein $\nu = 0.3$ and $\phi = 0.4$

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<th>$W[k]$</th>
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**Note**: Calculations by author. See also Table 2. In panel (b), mean unit shipping cost is 0.12.
Table 4  Model B: Cooperative farmers in a spatial economy

Outer boundary of farm at ring i from market
\[ x_i = \sqrt{i/(\pi g)} \] for \( 1 \leq i \leq N \) (4.1)

Mid radius for farm at ring i
\[ m_i = \sqrt{((i-0.5)/(\pi g))} \] for \( 1 \leq i \leq N \) (4.2)

Endowment for farm at ring i net of shipping cost
\( (1-f, 0) \) or \( (0, 1-f) \) for \( 1 \leq i \leq N \) (4.3)

Consumption of wheat and corn and sub-utility of farm with wheat endowment
\[ q_{11} = \nu(1-f) \]
\[ q_{12} = (1-\nu)(1-f)/P \]
\[ U_1 = (1-f)\nu(1-\nu)^{(1-\nu)/P^{1-\nu}} \] (4.4)

Consumption of wheat and corn and sub-utility of farm with corn endowment
\[ q_{21} = \nu(1-f)P \]
\[ q_{22} = (1-\nu)(1-f) \]
\[ U_2 = (1-f)\nu(1-\nu)^{(1-\nu)/P^{1-\nu}} \] (4.5)

Cooperative's balanced budget
\[ fN = \sum_i sm_i \] (4.6)

Minimum \( \beta \) for farmers participating in this co-operative
\[ \beta \geq (U[N-1]-U[N])/(V[N-1]-V[N]) \] (4.7)

Maximum \( \beta \) for farmers participating in this co-operative
\[ \beta \leq (U[N]-U[N+1])/(V[N]-V[N+1]) \] (4.8)

Notation

Givens (parameter or exogenous)
- \( g \): Density of farms (farms per square kilometer)
- \( N \): Number of farmers in market
- \( r \): Opportunity cost of land (assumed zero)
- \( \nu \): Risk aversion parameter
- \( \phi \): Exponent of wheat in utility function. Relative preference for wheat
- \( \phi \): Probability farmer has (0,1) endowment

Outcomes (endogenous)
- \( f \): Co-operative fee
- \( m_i \): Mid-radius of farm i
- \( N_1 \): Number of wheat farmers in market
- \( N_2 \): Number of corn farmers in market
- \( Q_1 \): Aggregate quantity of wheat offered in market
- \( q_1 \): Quantity of wheat consumed by farmer
- \( Q_2 \): Aggregate quantity of corn offered in market
- \( q_2 \): Quantity of corn consumed by farmer
- \( U \): Sub-utility of farmer
- \( V \): Variance
- \( x_i \): Outer boundary of farm i

Note: See also (1.1) through (1.6) I and Table 1.
Table 5  Model C: Competitive farmers and the market in a spatial economy where \( r \) is endogenous

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<thead>
<tr>
<th>Equation</th>
<th>Description</th>
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<tr>
<td>(5.1)</td>
<td>Outer boundary of farm at ring ( i ) from market ( x_i = \sqrt{(i/\pi g)} )</td>
</tr>
<tr>
<td>(5.2)</td>
<td>Mid distance for farm at ring ( i ) from market ( m_i = \sqrt{(i-0.5)/\pi g)} )</td>
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<td>(5.3)</td>
<td>Endowment for farm at ring ( i ) from market net of shipping cost ((1-\frac{r}{g \cdot s_m_i}, 0) ) or ((0, 1-\frac{r}{g \cdot s_m_i}))</td>
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<td>(5.4)</td>
<td>Rent at farm ( N ) at market boundary ( r[N] = 0 )</td>
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<tr>
<td>(5.5)</td>
<td>Rent at farm closer to market ( r[i] = s \sqrt{\frac{g}{\pi}} [\sqrt{(N-0.5)} - \sqrt{(i-0.5)}] )</td>
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</table>

Notation

Givens (parameter or exogenous)
- \( g \) Density of farms
- \( i \) position of farm: rings away from the market
- \( N \) Number of farmers in market
- \( \beta \) Risk aversion parameter
- \( \nu \) Exponent of wheat in utility function. Relative preference for wheat
- \( \phi \) Probability farmer has \((0,1)\) endowment

Outcomes (endogenous)
- \( m_i \) Mid-radius of farm \( i \)
- \( N_1 \) Number of wheat farmers in market
- \( N_2 \) Number of corn farmers in market
- \( Q_1 \) Aggregate quantity of wheat offered in market
- \( q_1 \) Quantity of wheat consumed by farmer
- \( Q_2 \) Aggregate quantity of corn offered in market
- \( q_2 \) Quantity of corn consumed by farmer
- \( r \) Market rent
- \( U \) Sub-utility of farmer
- \( V \) Variance
- \( x_i \) Outer boundary of farm \( i \)

Note  See also (1.1) through (1.6) I and Table 1.
Figure 1  Barter and Walrasian price setting in market with two farmers (A and B)

Note  \( \nu = 0.3 \). Initial endowments are \( q_A^1 = 0.40, q_A^2 = 0.60, q_B^1 = 0.60 \), and \( q_B^2 = 0.40 \). Indifference curve reached in absence of barter: \( abI_A \) for person A; \( afI_B \) for person B. Walrasian exchange. Horizontal axis scaled from 0.40 to 0.75; vertical axis from 0.40 to 0.65.
Figure 2  Conditional expected exchange rate (CEER) and size of market.

Note  $\alpha = 0.30$ and $\phi = 0.40$. Horizontal axis scaled from 0 to 30; vertical axis from 0 to 6.
Note $\alpha = 0.30$ and $\phi = 0.40$. Market size, $N$, shown as labeled dots for selected $N$ from 2 to 30. Horizontal axis scaled from 0 to 0.1; vertical axis from 0 to 0.45.
Figure 4  Models B and C: Maps of farms as rings in market of N6, N = 4, and N = 2.

(a) N = 6  
(b) N = 4  
(c) N = 2

Note
Figure 5  Model B: Shipping cost for marginal farmer and average shipping cost (cooperative fee) as function of the number of farmers in the market (N) when farms arranged as concentric rings around market

Note  
g = 30 and s = 0.40. Graph shown for 2 ≤ N < 30. Horizontal axis scaled from 0 to 35; vertical axis from 0 to 0.25.
Figure 6  Model B: $U[N]$, $V[N]$, and size of market in a spatial market: $\alpha = 0.30$, $\phi = 0.40$, $g = 30$, and $s = 0.40$.

Note  Market size, $N$, shown as dots for selected $N$ from 2 to 30. Gray line is the $U[N]$-$V[N]$ curve when shipping costs are zero. Black curve is the $U[N]$-$V[N]$ curve for a marginal individual taking into account higher shipping costs to reach larger market. Horizontal axis scaled from 0.04 to 0.10; vertical axis scaled from 0.2 to 0.5.
Figure 7  Model B: Risk and return by size of market.

Note $\alpha = 0.30$, $\phi = 0.40$, $g = 30$, and $s = 0.40$. Horizontal axis scaled from 0.065 to 0.072; vertical axis from 0.391 to 0.393.
Note \( \alpha = 0.30 \), \( \phi = 0.40 \), \( g = 30 \), and \( s = 0.40 \). Here, \( b \) and \( c \) are the lower and upper limits on \( \beta \) in Model B when the best size of market is \( N = 12 \): see Figure 9.7. For farmers for whom \( \beta = a \), best size of market is 10 in Model C and 14 in Model B. Horizontal axis scaled from 5 to 30; vertical axis from 0 to 3.
**Bibliography**


Footnotes