REGIONAL TRANSPORTATION SYSTEMS AND POPULATION IMPACT MODELLING: A THEORETICAL DEVELOPMENT

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Preface

This paper derives from an earlier report Ian Lord and John Miron prepared for this series (No. 2)—a report which was partially supported by the Joint Program. In this study, Miron extends the earlier model to incorporate, albeit in a fairly crude way, the role of non-transportation factors in urban growth. The theoretical development and the implications of the model are made explicit. It is hoped that this new formulation will further stimulate research on the role of transportation improvements on changing land use and population distributions.

R.D.M.

June 1972
1. The Need for Impact Modelling

The purpose of this paper is to develop and to examine a model which predicts the impact of a regional transportation system on the spatial distribution of the region's population. Such a model is a valuable tool for regional planning because of the effect of the spatial distribution of population on a region's ability to achieve a number of its important goals.

The basis for a specific population distribution goal might be in ecological, socioeconomic, ideological, or other considerations. An example of this is the Ontario Government's recent pronouncements regarding the Toronto-Centred Region (3,4). Here a major goal states that urbanization is to be encouraged in a linear strip along Lake Ontario. The primary basis for this goal appears to lie in an attempt to minimize long-run costs in the development of water and sewer services.

Consider the case where a region does not have a specific population distribution goal. Commonly, a government makes a general goal of developing 'adequate' public investment at minimum cost. In the case of a regional government, these public investments are usually in terms of utilities, schools, hospitals, libraries, fire halls, clinics, police stations, and day-care centres. A particular trait of all these facilities is that, spatially, they are user-oriented:
the notion of being 'adequate' includes that of locating
the investment close to the population being served. Thus,
a set of 'adequate' public investments can be made only if
the spatial pattern of population can be forecast. If a
transportation network affects the population distribution,
it is important to predict these effects far enough in ad­
vance so that investments can be planned and implemented.

Therefore, whether a region has a specific population
distribution goal or not, it is commonly interested in the
prediction of the impact of a transportation system on the
spatial distribution of population. In this paper, some
recent work in this area is reviewed and extensions are
made.

2. Schneider's Model of Access and Land Development

Schneider (5,6,7) presents a model which predicts the
impact of a transportation system on land development.
Land development is measured in terms of floorspace accu­
mulation.

Schneider suggests that the spatial distribution of
land-development represents an equilibrium. In this equil­
ibrium, land development at a site is related to the attract­
tiveness of that site and to the accessibility of that site
to all others in the region. The attractiveness of the
i'th site, \( R_i \), consists of the attractiveness of that site
at present, $R_{ai}$, and the amount of new floorspace accumulating over the projection period, $R_{fi}$.

$$R_i = R_{ai} + R_{fi} \quad i=1,2,\ldots,N \quad (1)$$

There are $N$ sites in the region. Note here that $R_{ai}$ is an unknown and that $R_{fi}$ is the variable we wish to predict.

Schneider uses a common definition for the accessibility at site 'i' of the sum of the attractivenesses of all sites in the region discounted by the functional distance, $f$, from 'i'.

$$I_i = \sum_{j} R_{fij} \quad i=1,2,\ldots,N \quad (2)$$

Note that the accessibility measure is the variable which reflects the nature of the transportation system.

Schneider also uses the common gravity model formulation to derive the number of trips from site 'i' to site 'j', $V_{ij}$, as a proportion of the total trips generated at 'i'.

$$V_{ij} = V_i R_{fij} I_j / I_i \quad i,j=1,2,\ldots,N \quad (3)$$

Finally, Schneider hypothesizes that the total number of trips generated at site 'i' is proportional to the amount of new floorspace accumulating at 'i' over the projection period.

$$V_i = \beta_0 R_{fi} \quad i=1,2,\ldots,N \quad (4)$$
Using the assumption that all trips are reciprocated and that functional distances are symmetrical, Schneider derives from the four equations above that

\[ R_{fi} = \frac{R_F a_i I_i}{(W - R_F I_i)} \quad i=1,2,\ldots,N \]  
(5)

where \( R_F \) is the total regional growth in floorspace

\[ R_F = \sum_j R_{fj} \]  
(6)

and \( W \) is the regional 'accessibility integral'.

\[ W = \sum_j I_j R_j \]  
(7)

A simple reformulation of equation (5) is possible and yields

\[ R_{fi} = \frac{R_i w_i}{W} \quad i=1,2,\ldots,N \]  
(8)

where \( w_i = R_i I_i \)  
(9)

Note now that \( W = \sum_j w_j \). Schneider's model thus states that the site 'i' gets a proportion of the total regional floor-space growth which is the same as the ratio of the product of its attractiveness and accessibility (\( w_i \)) to the sum of such products for all sites in the region (\( W \)).

This model has some elementary properties. First, the solution to (8) is independent of the scale of measurement of the functional distances. Multiplying the functional
distances by any constant leaves the solution to (8) unchanged. Thus, one need only derive relative, not absolute distances to solve (8).

Secondly, the solution to (8) is also independent of the scale of measurement for the attractiveness variable, $R_i$. Multiplying the attractiveness measure by a constant results in the solution value being also multiplied by the same constant. Thus, the model predicts a floorspace allocation in square feet which is exactly nine times the allocation in square yards. These two properties are elementary but necessary.

There are several unknowns in (8). The value for $R_{ai}$ is required in order to solve for $R_{fi}$ but is itself unknown. Suppose for some past period that we know $R_{fi}$, $R_F$, and the $f_{ij}$'s. Then (5) may be rewritten as the following

$$R_{ai} = R_{fi}(W - R_F I_i) / (R_F I_i) \quad i=1,2,...,N$$

which permits one to solve for $R_{ai}$. If estimates are available over several past periods, the future value for $R_{ai}$ can be extrapolated.

Another unknown in (8) is $R_F$, the total regional floor-space growth. This value has to be derived elsewhere and substituted into the model.

Finally, formulation (8) does not represent a simple set of equations which can be easily solved for $R_{fi}$ since
$R_{fi}$ enters into $R_i$, $I_i$, and $W$. These equations have to be solved iteratively for $R_{fi}$ and some problems may have no iterative solutions. In empirical work, it has been found that the existence of an iterative solution depends critically on the magnitude of $R_F$. As $R_F$ becomes large, there is an increasing likelihood that no iterative solution exists.

3. **Application of Schneider's Model to Population Impact Modelling**

In a recent paper, Lord and Miron (2) have made a few notational and conceptual changes to Schneider's model so that it can be used to predict population rather than floorspace impacts.

Let $G_i$ be the present population of site 'i', let $S_i$ be the net increment in population at site 'i' over the forecast period, and let $P_i$ be the population at site 'i' at the end of the forecast period.

$$P_i = G_i + S_i \quad i=1,2,...,N \quad (11)$$

Note that what has been done here is to merely replace $R_i$, $R_{ai}$, and $R_{fi}$ by $P_i$, $G_i$, and $S_i$ respectively. Carrying out this replacement in equations (1) through (8), it can be derived again that

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1 In a similar manner, equation (9) has to be solved iteratively for $R_{ai}$. 
$S_i = \frac{\Delta w_i}{W} \quad i=1,2,\ldots,N \quad (12)$

where $\Delta$ is the regional population increment,

$$\Delta = \sum_j S_j \quad (13)$$

$I_i$ is the accessibility of site 'i',

$$I_i = \sum_j P_{ij} f_{ij} \quad i=1,2,\ldots,N \quad (14)$$

and $w_i$ is defined by

$$w_i = I_i P_{i} \quad i=1,2,\ldots,N \quad (15)$$

This version of Schneider's model differs from the earlier one in one major respect. In the earlier case, it is necessary to predict the value of $R_{ai}$ before $R_{fi}$ could be forecast. In this later version, $G_i$ is merely the base-period population and does not need to be forecast.

A number of potential problems can be associated with formulation (12). These problems reflect the impression that this model is too mechanistic. There are few parameters and variables within this model which permit the adaptation of the model to the peculiarities of a study region. In an earlier application, it was found that growth predictions made, using the model, for historical periods were often far from the actual growth realized. 2

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2 Lord and Miron (2; pp 5-14).
4. Extensions to Schneider's Model

A first potential problem lies in the formulation for the accessibility variable. It is assumed, in equation (3), that accessibility determines the trip distribution pattern and, in equation (14) that accessibility is measured strictly in terms of proximity to population. However, a larger (smaller) number of trips are made to some sparsely (densely) populated areas than would be predicted by this formulation. This often occurs because of biases due to "amenities" or peculiarities (disamenities) of the areas. Some such amenities are public parks, solitude, points of public interest, particular economic mixes, etc. Each in its own way serves to attract (or repel) trip-makers in a manner not accountable for solely by the size of the population.

An extension to Schneider's model which allows for the problem of amenity biases is to redefine accessibility to be

\[ I_i = \sum_j X_j P_j f_{ij} \]

\[ V_{ij} = V_i X_j P_j f_{ij} / I_i \]

where \( X_i \) is the Amenity Bias Scaler (ABS) variable whose value is, as yet, undetermined. In a manner analogous to that used earlier, it can now be derived that, once, again,
\[ S_i = \frac{\Delta w_i}{W} \quad i=1,2,\ldots,N \]  \hspace{1cm} (18)

where \( w_i' \) is now given by

\[ w_i' = \sum_{i=1}^{N} X_i P_i \quad i=1,2,\ldots,N \]  \hspace{1cm} (19)

From formulation (18), it is seen that the site growth forecasts are independent of the scale of measurement of the ABS variable. Only the relative size of the ABS variable is important. Thus, there are an infinite number of sets of ABS values which produce the same set of site growth forecasts.

If, for some historical period, data are available on population growth, the end-period populations, and the functional distances in a region, then equation (18) can be re-expressed as

\[ X_i = \frac{S_i W}{(\Delta I_i P_i)} \quad i=1,2,\ldots,N \]  \hspace{1cm} (20)

This permits a solution for the ABS variable. Since \( X_i \) appears on both sides of this equation, it has to be solved for iteratively.

Suppose that data on population growth, end-period populations, and functional distances is available for several past periods. A historical series of ABS values for a particular site could be extrapolated using graphical or other methods. Two major problems are encountered.
First, since only relative ABS values matter and since the solution to (20) can be any of an infinite set of absolute values, it is advisable to introduce some overall constraint on the ABS values. A constraint might be

\[ \sum_{j} X_j = N \]  

(21)

or

\[ \sum_{j} X_j P_j = \sum_{j} P_j \]  

(22)

or

\[ \sum_{j} X_j P_j I_j = \sum_{j} P_j I_j \]  

(23)

Using these, derived ABS values for different periods can be regarded as consistent with one another in some sense.

A second major problem in using an ABS variable can be seen. The solution to (20) is a set of ABS values, one per site, which when substituted back into (18) permits a perfect prediction of some past population growth. Thus, the ABS variable is operationally akin to the residual in Multiple Regression. The danger is that the ABS value reflects not only amenity biases, but any number of other relevant variables and parameters not otherwise considered by the model. Thus, the estimation and extrapolation of ABS values has to be treated cautiously and in a manner which makes full use of any a priori information about the region and its transportation system.

Another potential problem with the model, as presented in equation (18), arises from the definition of the population
growth variable, $S_i$. One of the fundamental equations in Schneider's model states that the total number of trips generated at a site is proportional to the incremental population:

$$V_i = \beta_0 S_i, \quad i=1,2,...,N$$

Thus, aggregate trip generation will be negative unless the incremental population, $S_i$, is positive. Since the number of trips must be positive, a numerical solution which calls for a negative population growth is infeasible. However, populations can and do decrease at some sites in a region and the problem is to redefine the model so that the population at a site can decline while allowing for positive trip production.

An extension to Schneider's model which allows for the problem of declining site populations is to define a new variable, $C_i$, which is the Latent Mobility Potential (LMP) of site 'i'. The Latent Mobility Potential of a site is defined to be the proportion of the base-year population which is expected to move, either within the site or from site to site, over the projection period.

The LMP of a site is a function of the social, demographic, and economic characteristics of the base population of the site. Researchers have found that LMP is dependent on the age, marital status, and income structure of the
population. Since these variables are not considered elsewhere in the model, it is assumed that values for LMP are defined outside the Schneider model and merely represent an additional data input. However, future research might well be concerned with the fusion of Schneider's model with predictive models of LMP.

Let us define $P_{it}$ to be the population of site 'i' at time 't'. Redefine $S_i$ and $G_i$ as follows.

$$S_i = P_{it} - P_{it-1} + C_i P_{it-1} \quad i=1,2,\ldots,N \quad (25)$$

$$G_i = (1 - C_i)P_{it-1} \quad i=1,2,\ldots,N \quad (26)$$

$S_i$ is now defined to be the net population growth plus the LMP portion of the base-year population. $G_i$ is now the non-LMP proportion of the base-year population. $G_i$ can be termed the 'fixed' population because it is the number of persons who are not expected to move over the projection period. In the same sense, $S_i$ might be termed the 'floating' population. Computationally, $S_i$ might be negative although this is not very likely if $C_i$ is defined correctly.

Except for these changes in the definitions of $S_i$ and $G_i$, there are no other changes in the formulation of Schneider's model so that equation (18) still holds. Conceptually this is a different problem however. Instead of

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3 See Wolpert (8).
allocating net regional growth in population among the sites, equation (18) now allocates the regional floating population, a somewhat larger amount. Also, it is now possible for \( P_{it} \) to be less than \( P_{it-1} \) while \( S_i \) is still positive.

The two extensions discussed so far, the ABS and LMP variables, have been implemented in a study of the Toronto-Centred Region which has been carried out simultaneously with the work recorded here.\(^4\) Considerable success has been encountered in this empirical application with the use of a constant value for the LMP variable for all sites.\(^5\) However, it was noted that, by varying the constant value of the LMP variable, there is a major relationship between the value of the LMP variable and the derived ABS values.

This observed relationship between the LMP and ABS variables leads to an interesting question. If it is assumed that the LMP variable takes on a constant value for all sites and that the ABS variable is constrained to sum to some fixed value as in (21), can one determine the true value for the LMP variable from the effect it has on the distribution of the ABS variable? As a more specific question, can one validly select that value for the LMP variable which minimizes the standard deviation of the constrained ABS value set?

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4 Lord (1).

5 Wolpert (8) finds that, over a five-year period, approximately one-half of all persons in the U.S.A. change their residences. This value of 0.5 is adopted for five-year spans in the empirical application.
It is not possible at present to give a theoretically-valid response to these questions. However, the empirical application has been encouraging in this area. From the work of Wolpert (8), it is suggested that the LMP variable currently takes a value of about 0.5 over a five-year period. In the study region examined, there is generally a minimization of the standard deviation of the ABS variable values around this particular LMP value. This is an early indication that the interaction of the LMP and ABS variables may provide an estimate of the magnitude of the LMP variable in the absence of any other kind of information.

5. A Generalization of Schneider's Model

Equation (24), which states that the total number of trips generated at a site is proportional to $S_i$, causes some theoretical concern. Of all the equations hypothesized by Schneider, this is the single most contentious one. A more generalized formulation of this equation would allow for the effect of floating population, fixed population, and accessibility on the total number of trips produced.

$$V_i = \beta_0 S_i^{\gamma_1} G_i^{\gamma_2} I_i^{\gamma_3} \quad i=1,2,\ldots,N$$  \hspace{1cm} (27)

Equation (27) is a log-linear model of trip production which takes all three of these variables into account in predicting the total number of trips generated.
Secondly, there are the assumptions that all trips are reciprocated and that functional distances are symmetrical.

\[ V_{ij} = V_{ji} \quad \text{and} \quad f_{ij} = f_{ji} \quad i,j=1,2,\ldots,N \quad (30) \]

Using equations (27) through (30), it can be derived that

\[ s_i = \frac{\Delta w_i}{W} \quad i=1,2,\ldots,N \quad (31) \]

where

\[ w_i = (P_i t X_i)^{B_1} I_i^{B_2} G_i^{B_3} \quad i=1,2,\ldots,N \quad (32) \]

and

\[ B_1 = 1/\gamma_1, \quad B_2 = (1 - \gamma_3)/\gamma_1, \quad B_3 = -\gamma_2/\gamma_1 \quad (33) \]

This generalization of Schneider's model is fundamentally different from any of the earlier extensions and from Schneider's own original model. With the addition of the three elasticities, each with a specific theoretical interpretation, some relevant socioeconomic characteristics of the population of the region can be specifically and particularly allowed for. This has not been possible with this model until this point.

Another important difference in this last model is that historical data can now be used to derive estimates for each of the three elasticities. Since these trip estimations procedures are discussed in Section 7 below.
generation elasticities have an interpretation outside the context of this model, these historical estimates can be compared with elasticity values derived from other kinds of transportation studies. If the elasticity estimates are not similar, this would imply that either this generalized version of Schneider's model is mis-specified or that the other kinds of studies have been done incorrectly. In either case, much information should be gained.

6. Properties of the Generalized Model

The generalized version of Schneider's model can be manipulated into forms other than that presented in equation (31). These forms permit new interpretations of and insights into the model.

It is possible to re-arrange the variables in equation (31), without making any new assumptions or hypotheses, to derive that

$$S_i = \frac{\Delta_i W_i}{W_i} \quad i=1,2,\ldots,N$$

(34)

where $\Delta_i$ is the floating population allocation of all other sites in the region,

$$\Delta_i = \Delta - S_i \quad i=1,2,\ldots,N$$

(35)

and $W_i$ is the accessibility integral defined for all other sites in the region.
Several implications can be drawn from equation (34). First, a comparison of (34) with (31) yields the conclusion that

\[ \frac{\Delta_i}{W_i} = \Delta/W = \frac{\Delta_j}{W_j} \quad i, j = 1, 2, \ldots, N \]  

(37)

Schneider (6; p 150) refers to the reciprocals of these ratios as 'regional temperature' drawing out the analogy between the regional equalization of \( \frac{\Delta_i}{W_i} \) in the model and the equalization of temperature in entropy theory in thermodynamics. Schneider also suggests that a useful method for delineating regions is to measure regional temperature at various sites over some historical period. Regions or subregions are identified by abrupt changes in regional temperature at neighbouring sites. This approach offers some promise although there are some difficult methodological and operational problems associated with its implementation.

A second implication of equation (34) is that \( S_i \) will be zero if \( \Delta_i \) is zero. Thus, it is impossible to have one site with a growing population if the rest of the population is stagnant (when \( C_i \) is zero for all sites) or declining (when \( C_i \) is nonzero for all sites). However, if, for any other site 'j', \( S_j \) is positive, then \( S_i \) may also be positive. Note also however that \( S_i \) will be zero if \( X_i, P_{i t}, G_i, \) or \( I_i \) is zero.
An examination of equation (32) reveals the significance of the constant \( \beta_2 \) in this generalized model. If \( \beta_2 \) equals zero, which occurs when \( \beta_3 \) equals unity, then accessibility plays no role in determining the population allocation, \( S_i \), at each site. In turn, this means that, when \( \beta_2 \) equals zero, the transportation system has no effect on the population distribution. Earlier versions of the model presume a role for the transportation system by setting \( \beta_2 \) at a value of unity.

An examination of the two site region provides some basic insights into the logic of the model. In the two site model, equation (34) can be re-expressed as

\[
S_i = S_2 \left[ \left( \frac{x_1 p_{1t}}{x_2 p_{2t}} \right)^{\frac{1}{\gamma_1}} \left( \frac{I_1}{I_2} \right)^{\left(1 - \gamma_3\right)} \left( \frac{G_1}{G_2} \right)^{-\gamma_2} \right] \quad (38)
\]

If it is assumed that \( f_{12} \) is close to zero, relative to \( f_{11} \) and \( f_{22} \), this model then can be rewritten as

\[
\frac{S_1}{p_{1t}} = \frac{S_2}{p_{2t}} \left[ \left( \frac{x_1}{x_2} \right)^{\frac{2 - \gamma_3}{\gamma_1}} \left( \frac{f_{11}}{f_{22}} \right)^{\frac{1 - \gamma_3}{\gamma_1}} \left( \frac{p_{1t}}{p_{2t}} \right)^{\frac{2 - \gamma_1 - \gamma_3}{\gamma_1}} \left( \frac{G_1}{G_2} \right)^{-\gamma_2} \right] \quad (39)
\]
If it is assumed further that the elasticity of intrasite functional distance with respect to population is negative unity, or in other terms,

\[ f_{11}^{P_{it}} = a \quad \text{and} \quad f_{22}^{P_{2t}} = a, \]  

(40)

then (39) becomes

\[
\frac{S_1}{P_{1t}} = \frac{S_2}{P_{2t}} \left[ \left( \frac{x_1}{x_2} \right)^{2\gamma_3} \left( \frac{f_{11}}{f_{22}} \right)^{\gamma_1 - 1} \left( \frac{G_1}{G_2} \right)^{-\gamma_2} \right] \]  

(41)

Thus, the growth rates for each site are in fixed proportion, the proportion varying with the ratios of fixed populations, intrasite functional distances, and ABS values. In the earlier versions of Schneider's model, where \( \gamma_1 \) is unity and \( \gamma_2 \) and \( \gamma_3 \) are zero, this formulation reduces to

\[
\frac{S_1}{P_{1t}} = \frac{S_2}{P_{2t}} \left[ \left( \frac{x_1}{x_2} \right)^2 \right] \]  

(42)

In this case, if site '2' has twice the ABS value of site '1', site '2' will have four times the growth rate of '1'.

The three site and higher order models have proved to be too difficult to handle mathematically. It might well be that sensitivity analysis applied to specific numerical examples represents the only future line for research here.
7. **Empirical Application**

The problem of estimating values for the three elasticities is to be discussed in this section. All data used in this section are taken from Lord (1).

Suppose that, for some historical period, data are available for fixed population, floating population, functional distances, and the ABS variable for each of N sites in a region. One wishes to estimate values for the parameters \( \gamma_1, \gamma_2, \) and \( \gamma_3 \).

Using \( \ast \) to denote a natural logarithm, (31) can be rewritten as

\[
S_i^\ast + W^\ast - \Delta^\ast = \beta_1 (P_i t X_i)^\ast + \beta_2 (I_i)^\ast + \beta_3 (G_i)^\ast \quad (43)
\]

\[
i=1,2,\ldots,N
\]

Formulation (43) suggests that Ordinary Least Squares can be used to estimate the parameters \( \beta_1, \beta_2, \) and \( \beta_3 \). However, \( W^\ast \) presumes knowledge of these three parameters. This means that a version of nonlinear least squares must be used in this case. The following algorithm is suggested for this problem and this presumes the use of an interactive computer terminal.\(^8\)

\[^8\] An APL/360 compiler system was used here.
(i) Choose initial estimates of the three parameters. 
(ii) Using these parameter estimates, calculate a new variable, $Z_i$, given by

$$Z_i = S_i^* + W^* - \Delta^*$$

$$i = 1, 2, \ldots, N$$

(iii) Use Ordinary Least Squares, with constant constrained to zero, to re-estimate the three parameters by regressing $Z_i$ against $(P_{it}X_i)^*$, $I_i^*$, and $G_i^*$. 
(iv) If the re-estimated parameter values are the same as those used in step (ii), the solution has been found. 
(v) Otherwise, use a decision rule to derive new parameter estimates. 
(vi) Go to step (ii).

In the empirical work undertaken here, no simple decision rule appears to be particularly relevant in step (v). In each application, an intuitive approach together with much trial-and-error resulted in success. When a solutions set of $\beta$'s is obtained, equation (33) is used to derive estimates of the $\gamma$'s. Estimates derived using this procedure in conjunction with data relating to the Toronto-Centred Region are presented in Table 1. 

Because the estimation of these elasticities requires the use of nonlinear least squares, there are no valid measures of goodness-of-fit available. In Table 1 is presented the standard error of estimate for equation (43) although its value is purely descriptive.
TABLE 1
Estimates of trip elasticities by census intervals for the Toronto-Centred Region, 1931 to 1971

<table>
<thead>
<tr>
<th>Years</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>Standard Error of Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1931 to 1941</td>
<td>0.25</td>
<td>0.75</td>
<td>1.55</td>
<td>0.516</td>
</tr>
<tr>
<td>1941 to 1951</td>
<td>0.36</td>
<td>0.65</td>
<td>1.72</td>
<td>0.385</td>
</tr>
<tr>
<td>1951 to 1956</td>
<td>0.38</td>
<td>0.61</td>
<td>1.80</td>
<td>0.403</td>
</tr>
<tr>
<td>1956 to 1961</td>
<td>0.45</td>
<td>0.54</td>
<td>1.68</td>
<td>0.273</td>
</tr>
<tr>
<td>1961 to 1966</td>
<td>0.51</td>
<td>0.51</td>
<td>2.02</td>
<td>0.450</td>
</tr>
<tr>
<td>1966 to 1971</td>
<td>0.41</td>
<td>0.57</td>
<td>1.66</td>
<td>0.339</td>
</tr>
</tbody>
</table>

SOURCE: See text.

Interesting interpretations can be made of the time trends in Table 1. The elasticity of trip generation with respect to floating population, $S_j$, is smaller than its counterpart for fixed population. It increased in value steadily over the period of 1931 to 1966 and then fell sharply to 1971. There may be a link here between this elasticity and the business cycle.

A comparison of the elasticities of floating and fixed populations, especially in earlier periods, leads one to suspect that the floating population, is characterized by much lower incomes since their trip generation
elasticity is quite small. However, their elasticity increased steadily, likely as a result of increased incomes, until 1966 when it is equal to that of the fixed population. The fall from 1966 to 1971 can be explained by the recession, occurring during this period, which may have hit the income level of the floating population harder than that of the fixed population.

Another somewhat surprising feature about the floating and fixed population elasticities is that they sum almost perfectly to unity over the whole of the period of 1931 to 1971. Thus, increasing the floating and fixed population by the same constant ratio, \( r \), will result in an increased number of generated trips by the same ratio, \( r \).

Finally, the elasticity of trip generation with respect to accessibility appears to be increasing over time. However, setbacks occur in the 1956-1961 and 1966-1971 periods. This might well be due to the business cycle also as both of these periods include business recessions.

The reasonableness of the derived estimates suggests that this generalized model of population impacts is quite valid. Also, the size of the derived estimates suggests that Schneider's model, as presented in the earlier versions, may be an inappropriate one to use without the generalized parameters.

A final note is necessary about the estimation of the ABS variable in the context of the generalized model. In
the empirical application above, it is assumed that all $X$ values are unity. If some a priori information is available about this variable, such data might usefully be included at the outset of the estimation procedure for the $\gamma$'s. However, it is assumed here that the $ABS$ variable continues to take on a 'residual' role until better estimation procedures are available for it. In this light, it is initially assumed that all $X_i$'s have a unit value. Using this, the $\gamma$'s are estimated and these estimates are then used to solve for a set of $X_i$'s given by

$$X_i = \left[ \frac{I_i (\gamma_3 - 1) \quad p_{i1}^{\gamma_2}}{p_{it}^{\gamma_2}} \right] \left[ \frac{W S_i}{\Delta} \right]^{1/\gamma_1} \quad i=1,2,\ldots,N \quad (45)$$

8. Conclusion

In this paper, some recent developments in the modelling of population impacts have been presented. A generalized model which predicts the spatial distribution of population associated with a particular transportation network has been outlined. A small empirical test has been undertaken to illustrate that the central hypotheses of this generalized model are valid. Finally, some of the estimation problems involved in this model have been discussed.

It is concluded that this model offers much promise as an operational, theoretically-valid tool for use by regional planners.
BIBLIOGRAPHY


(2) Lord, I. J. and J. R. Miron, Modelling the Impact of Proposed Transportation Networks on the Toronto-Centred Region, Research Paper 45 for the Centre for Urban and Community Studies, University of Toronto and Research Paper 2 for the University of Toronto-York Joint Program in Transportation, (Mimeo., 1971).

(3) Ontario Government, Design for Development: The Toronto-Centred Region (Queen's Printer, 1970).


