

Ellipses on the surface of a picture

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RUNNING HEAD: Perspective and ellipses

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## Abstract

Shapes on the surface of a perspective picture may be misperceived. Subjects picked a match for an ellipse depicting the circular top of a cylinder. The top was depicted as tilted from  $5^{\circ}$  to  $85^{\circ}$  around a horizontal axis, generating a series of ellipses on the picture surface. The matches were biased towards a circle over a wide range of midrange tilts, which suggests they were seen as in-between the shape on the surface and the shape they depicted.

Ellipses on a picture’s surface

When we are drawing, we often begin by outlining an area on the picture surface to match a shape in the scene. Of interest here is that the area on the surface will often change its appearance as soon as a perspective pattern is added to show the depth in the scene (Juricevic and Kennedy, 2006; Kennedy, Juricevic, Hammad and Rajani, in press).

In the present study, we consider an elliptical area on the picture surface and how it looks when it depicts the top of a tilted cylinder drawn in perspective. We ask subjects to pick the correct match for the ellipse from a set of ellipses (Figures 1 and 2). We ask whether the match is biased towards the ratio of a circle, as several theories predict, and how errors are related to the cylinder’s tilt. Does bias arise at extreme tilts of  $5^{\circ}$  and  $85^{\circ}$ , at moderate tilts of  $45^{\circ}$ , or across the range of tilt angles?

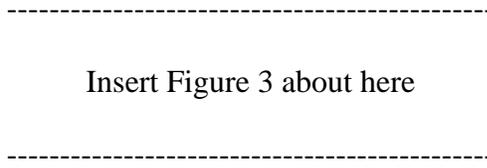
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Perspective and a picture surface

Geometrical illusions in which shapes on surfaces are reliably misperceived may often be due to perspective and mis-applied size constancy (Gregory, 1972; 1980). They have been of interest since the development of linear perspective in the 1400’s and have been debated at length in Gestalt and ecological theory of pictures (Arnheim, 1954; Gibson, 1979).

To focus attention on the topic of interest it may be helpful to consider two examples, one to do with angles and the other with “table tops”.

May observers misperceive angles in Figure 3, which is based on a sketch in Kennedy & Juricevic (2006). In Figure 3, two perspective drawings of cubes have quadrilateral tops. In the nine quadrilaterals below are the matches for the tops. Most observers are surprised to learn that the match for the top of the left cube drawing is row 3, column 3 and for the right cube is row 2, column 2. In the cube drawing on the left the obtuse top-most internal angle is  $143^{\circ}$ . Many observers suggest its closest match is found at row 1 column 1, though that is  $130^{\circ}$ . Matching its acute angle ( $41^{\circ}$ ) with row 1, column 1's acute angle ( $54^{\circ}$ ) is an overestimation of  $13^{\circ}$ , almost one-third in error.



An effect of perspective on shape perception occurs in the Shepard (1990) “table tops” illusion. Table-top shapes are congruent on the picture surface but do not look it. One is foreshortened along its length and the other across its width. What is notable for the present purposes is that the foreshortening not only changes the dimensions of the depicted table-tops, it affects the apparent sizes of the shapes on the picture surface.

Projection from a 3-D scene to a 2-D surface

In Figure 1, cylinders are drawn on a 2D surface with foreshortening of the circular top and the lines representing the height of the cylinder indicating the cylinder is tilted. The cylinders are drawn in one-point “linear” projection. The top and the bottom of each cylinder project as ellipses. The left and right sides of each drawing converge towards the bottom ellipse. The convergence indicates tilt (see Figure 4).

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Consider a highly extreme ellipse in Figure 1. The information it supplies directly is about its own aspect ratio. That is, the oval lines in a picture of a cylinder allow “direct” perception of their own aspect ratio, and in addition the “indirect” perception of circular tops and bottoms of a depicted object (Gibson, 1979; Arnheim, 1954). We test direct perception of the narrow ellipse on the picture surface to discover how it is affected by the lines allowing the indirect perception of the circular form.

We use a matching task here to avoid subjects having to report ratios as numbers, and to permit subjects to report that two forms simply look alike.

Projective, Classification, Good-form and Perspective-features theories

Arnheim (1954) and Sedgwick & Nicholls (1993) suggest “crosstalk” between the optic information specifying a depicted form and the form’s projection on a picture surface gives rise to “in-between” percepts that are neither veridical about the picture surface area nor full-constancy impressions of the depicted object (Sedgwick, 2003). Crosstalk occurs from the pictured world to the picture surface and vice versa. So far as judging a projected form is concerned, where might crosstalk be concentrated -- at extreme tilts, medium tilts or across the range?

A “projective” hypothesis about crosstalk is that as the projections on the surface deviate from the depicted form, bias increases. That is, tilts of 5 and 15 degrees, at which cylinder tops project slim ellipses, entail large illusions, middling tilts (45 degrees) less and at 75 and 85 degrees there is very little crosstalk since the cylinder tops are close to

parallel to the picture surface and almost project their true forms. In this theory, only extreme foreshortening triggers strong perceptual corrections.

Alternatively, it may be precisely the cases in which ellipses approach the top's form that boost crosstalk. Perhaps in these conditions the ellipse can appear perfectly circular. In a "classification" theory, top-down effects are said to arise from categorizing stimuli as showing objects. Close to an aspect ratio of 1, an ellipse categorized as showing a cylinder's circular top might be seen as a perfect circle. Categorization effects might be minimal for aspect ratios far from 1, this theory could contend, since they depart too much from the shape of the familiar object to be made into perfect circles.

A "good form" theory might predict substantial errors not only for ellipses close to circular but also for slim ellipses. If physical conditions approach a "simpler" shape, a percept with full symmetry and equality of its parts is favoured, Gestaltists argue. When the differences from a good form are modest, "projective distortions are observed only by the select few who have been trained to watch out for them" (Arnheim, 1986). A possible implication of "good form" theory is that a very slim ellipse might be seen as close to a straight line, since a straight line is another good form. That is, very slim ellipses might be seen as even slimmer than is correct. In this "good-form Gestalt" theory ellipses projected by both large (e.g. 85 degrees) and small (e.g. 5 degrees) tilt angles will suffer the most correction. Intermediate angles (45 degrees) will incur less.

A perspective-features theory could argue that at tilts close to 45 degrees the lines on the page showing convergence and foreshortening are most evident. The lines for the cylinder's sides are long enough to converge distinctly at intermediate angles. At 5 and 15 degrees tilt, the lines are long but their convergence is minimal. At a tilt close to 85

degrees, convergence is present but the lines are too short and stubby for it to be obvious. If so, lines on the page with perspective features most readily offer crosstalk at intermediate tilts.

In sum, to test 4 hypotheses about the tilts creating errors, we ask subjects to match ellipses in a set of drawings of a tilted cylinder with ellipses in a set of ellipses.

## Method

### Participants

Eleven participants (8 female, mean age 19, range 4 years) were recruited from a first year psychology class at the University of Toronto at Scarborough. Each had normal or corrected-to-normal vision and was naïve to the purpose of the experiment.

### Stimuli

All stimuli were generated in a bespoke program called Angle Grinder, written in Java programming language and using the “Java 3D API” to render the cylinders. They were presented to all participants on a 15” computer monitor. Cylinders were drawn in linear perspective with white lines 1mm thick on a black background. They were designed as “wire-frame” presentations, rather than “solid” opaque objects, showing the upper-most and lower-most ellipses, as well as the two upright lines depicting the body of the cylinder.

The top face was tilted from the horizontal in 10° increments between 5° and 85°. At 5° the ellipses were highly foreshortened. At 85° the projections of the ellipses were very close to a circle. Each tilt was a rotation out of the picture plane about the nearest point of the upper-most ellipse, which was fixed at eye level to the observer. At a 90° tilt the top would have been projected as a straight line. As a result of this manipulation the

height of the upper-most ellipse (as measured between the lowest and highest points) varied from 3mm (5° tilt) to 34mm (85° tilt).

Each participant was shown the 9 cylinder drawings ten times in a random order, with no two participants receiving the same order, in two blocks; in one, the cylinder was present, and the other had only the cylinder's upper-most ellipse (Figure 5). Half of the subjects were presented with the cylinder-drawings first, and half were given the ellipses-alone first.

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 Insert Figure 5 about here  
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Procedure

Participants sat approximately 50cm from the computer monitor. In one order, they were first told that they would be presented with ellipses of different sizes on the computer screen (ellipse-only condition), and were shown examples of ellipses taken from actual stimuli (i.e., the tops of the depicted cylinders). They were to use the mouse pointer to click on the matching ellipse at the lower half of the screen. The order of ellipses on the lower half of the screen was randomized with each presentation.

After 10 presentations of each ellipse (90 trials), instructions were given for the second block (also 90 trials). In this, participants were informed that they would be presented with two ellipses, one above the other and connected at the sides by two lines (cylinder condition). They were instructed to use the mouse pointer to match the upper-most ellipse at the top of the screen with the corresponding ellipse at the lower half of the

screen. As noted above, half the participants received the ellipse-only condition first, and half received the cylinder condition first.

It is important to note that the instructions ask subjects to match the ellipses, not the cylinder that is depicted, in describing the stimuli. The depicted cylinder has a circular top, no matter what the tilt. The ellipse on the screen varies from trial to trial. The instructions ran as follows: “You will now be presented with two ellipses on the computer screen, joined at their edges by two straight lines. The ellipses you see will be of different sizes. Some may be short and some may be tall. In some cases the two ellipses may overlap.” The aim was to have participants to judge the upper-most 2D ellipses that vary in shape, rather than the constant 3D object depicted in the display, and to match the ellipse that is present on a given trial to the corresponding shape at the lower half of the screen.

### Results

The height of each ellipse presented, subtracted from the participants mean estimations, results in mm values about 0. Overestimations are positive. The mean responses and errors for both conditions are represented in Figures 7 and 8. For significance, an alpha level of 0.05 was applied to all comparisons.

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 Insert Figures 7 and 8 about here  
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Cylinder-top ellipses were larger than ellipses-alone on a 2 (ellipse vs. cylinder) X 9 (presented ellipse height mm: 3, 8, 13, 18, 22, 26, 30, 32, 34) within-subjects ANOVA,  $F(1, 8) = 29.3$ ,  $MSE = 119.2$ . Cylinder-top ellipses were judged different from

ellipses-alone only at midrange tilt angles (main effect  $F(8, 64) = 29.2$ ,  $MSE = 45.0$ ; paired-sample T-tests different at angles of:  $25^\circ/13\text{mm}$  ( $t(10) = 4.53$ ),  $35^\circ/18\text{mm}$  ( $t(10) = 5.79$ ),  $45^\circ/22\text{mm}$  ( $t(10) = 6.50$ ),  $55^\circ/26\text{mm}$  ( $t(10) = 3.66$ ). Hence there was a significant interaction between cylinder present or absent and tilt angle ( $F(8, 64) = 17.2$ ,  $MSE = 21.9$ ).

Cylinder top-faces were judged greater than ellipses on their own, with differences ranging from  $-1.9\text{mm}$  (at  $85^\circ$  tilt) to  $4.6\text{mm}$  (at  $25^\circ$  tilt), a total of  $6.5\text{mm}$ . For ellipses-alone, the range is small, between  $-1.3\text{mm}$  (at  $85^\circ$  tilt) and  $1.1\text{mm}$  (at  $25^\circ$  tilt), a total of  $2.4\text{mm}$ .

In sum, ellipses depicting cylinder tops are judged larger than ellipses on their own at intermediate tilts.

### Discussion

Errors were most evident from  $25^\circ$  to  $55^\circ$ . The errors are overestimations. A theory about perspective's features is supported rather than projection, good-form or classification hypotheses.

Contrary to the projection hypothesis, the largest errors were not for the slimmest ellipses. Good-form Gestalt theory fails since the biases were not most evident at extreme tilts. Classification theory expected the largest errors to occur as ellipses approach the circular form of the depicted object, but they were at intermediate angles.

Since the test and matched ellipses were presented simultaneously, it seems unlikely that subjects forgot the ellipse's form and then made judgements on the basis on the depicted form. Indeed, if they had forgotten, all the judgments would have been of circular forms. (We suggest that errors might grow if the time between exposure and

matching was expanded and distracting tasks intervened, but memory effects like these are subsequent to perception, our primary focus.)

Errors concentrated at middling values support the perspective-features hypothesis. Convergence and foreshortening are shown most clearly on the picture surface at these intermediate angles, whereas tilts close to  $0^\circ$  and  $90^\circ$  incur short lines or minimum convergence.

Perspective has many consequences for perception. Physically, it controls the angles subtended by objects at the observer's vantage point and the projections onto a picture surface. Psychologically, it is the geometry that vision must approximate in dealing with many constancy problems, and needs to deal with in looking at pictorial surfaces. The present study indicates – in terms from Gregory (1972) – that vision misapplies perspective features to the forms it projects onto pictorial flat surfaces. On the one hand, vision is using perspective in looking at surfaces of any kind, and on the other hand prominent perspective features drawn on the surface bias observers judging features of the surface itself.

Logically, in normal environmental circumstances perspective may apply even handedly, without bias. This would occur if converging lines on textured surfaces were evenly distributed. But a textured surface projects a gradient to the observer's vantage point. That is, the evenly-distributed texture on the surface projects optic texture that has a bias. Vision's task is to use the bias in the gradient to have the observer see the slant of the surface and the true form of parts of the surface. In pictures, the gradient is presented on a surface. That is, the markings on the surface contain a bias – features to do with perspective. Vision's task is now to discern the true shape despite the bias. The results of

our experiment suggest the task of discounting the bias from perspective features is not solved perfectly.

Our results suggest as many questions as they answer. For example, if the cylinder was transformed into a truncated cone, with the top ellipse at the high, narrow end of the cone, convergence would be more obvious at lower tilts than for cylinders. In contrast, it would be most obvious for higher tilts if the top was at the cone's broader end. If the cones were combined like some beer glasses that have a slim waist, the convergence of the lines connecting to the top would be different from the lines connecting to the bottom ellipse. This shape could help distinguish information for tilt from effects simply due to converging lines. Indeed, perhaps the trickiest issues here are to do with the relation between high-level perspective information, which is about depth and pictured shape, and flat features which may produce low-level pattern effects in vision irrespective of the high-level information. We have used tilt information here in the present study, and the effects are consistent with moderate crosstalk from foreshortening effects, but in principle it is hard to distinguish possible foreshortening from effects from the converging lines that carry information about foreshortening. However, one advantage of the theory of misattribution of perspective is that it indicates what geometrical variables to explore systematically – tilt, foreshortening, convergence and other aspects of perspective. If these have crosstalk effects in accord with the spatial depth and slant they signify then the perspective theory is useful indeed.

Figures 1 and 3 depict highly familiar symmetrical forms. The perspective-features theory of crosstalk should apply broadly, for example to unfamiliar and asymmetrical forms. Figure 1 allows observations to be biased towards a circle, and the

bias for Figure 3 is a square. But other forms could be inscribed within these e.g. a scalene or isosceles triangle, or an irregular pentagon. If the perspective-features theory is correct, the angles that subjects perceive the depicted referents having should be the angles to which the shape on the surface appears biased.

In conclusion, observers' perceptions of ellipses on the surface of a perspective picture are biased towards the referent of the ellipses.

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### Acknowledgments

We thank high-school science teachers Seth Bernstein and Joe Hockin (members of TSTOP, Ontario, 2006) for comments on theories of the angle illusion. We also offer our appreciation to Sandacre Technology for their programming expertise.

## Figure Captions

Figure 1. Orientation of cylinder stimuli through nine tilt angles from the horizontal.

Figure 2. Test condition with a cylinder depicted, and 9 ellipses to match to the cylinder's top ellipse. Many observers match the cylinder top with the ellipse in the upper row, third from the left, though the correct match is upper row, leftmost ellipse.

Figure 3. Two of the quadrilaterals correspond to the top faces of the two cubes, but observers misidentify the matches. The correct matches are the middle and bottom-right quadrilaterals.

Figure 4. The extended lines converge in one-point perspective, indicating that the cylinder is tilted ( $45^{\circ}$  from horizontal).

Figure 5. Test condition with an ellipse at the top and a set of 9 ellipses below

Figure 1.

$5^{\circ}/15^{\circ}/25^{\circ}$

$35^{\circ}/45^{\circ}/55^{\circ}$

$65^{\circ}/75^{\circ}/85^{\circ}$

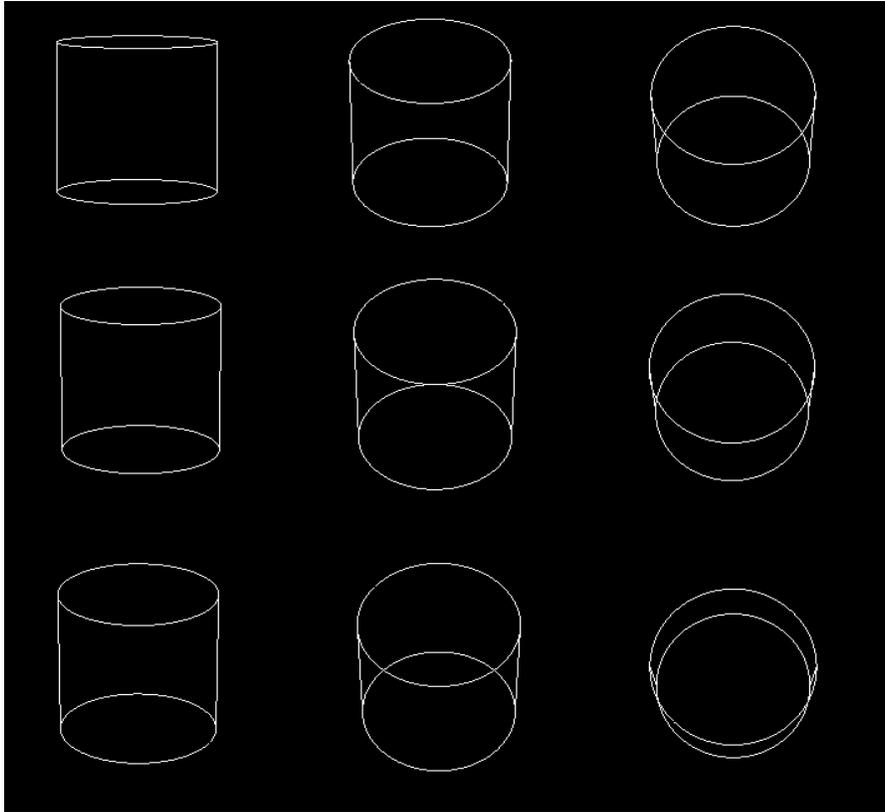


Figure 2.

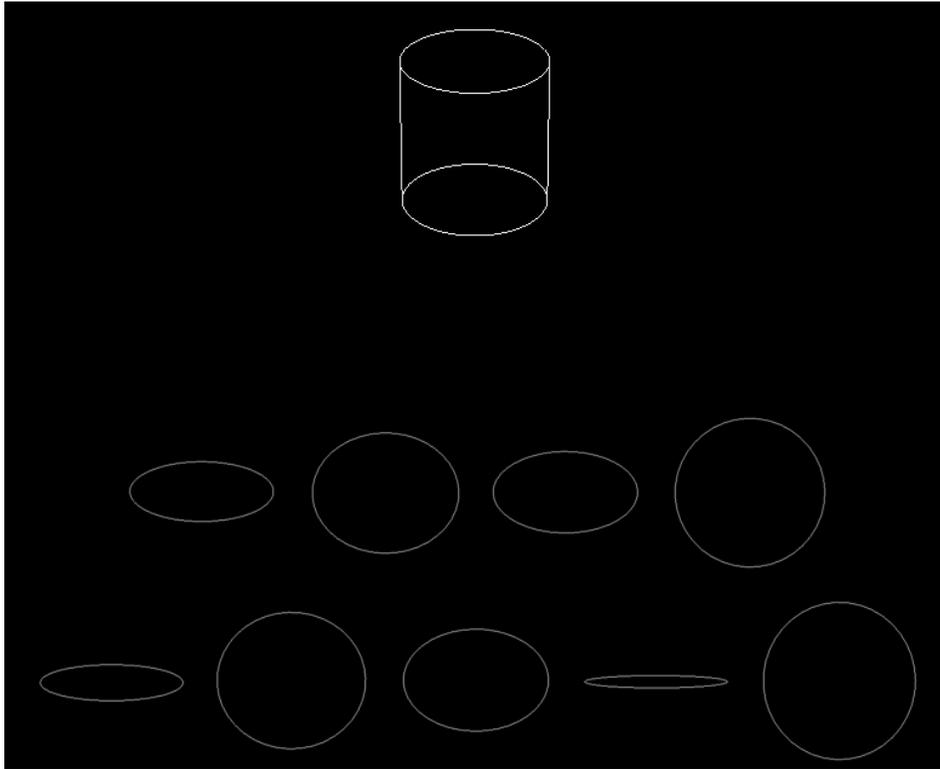


Figure 3.

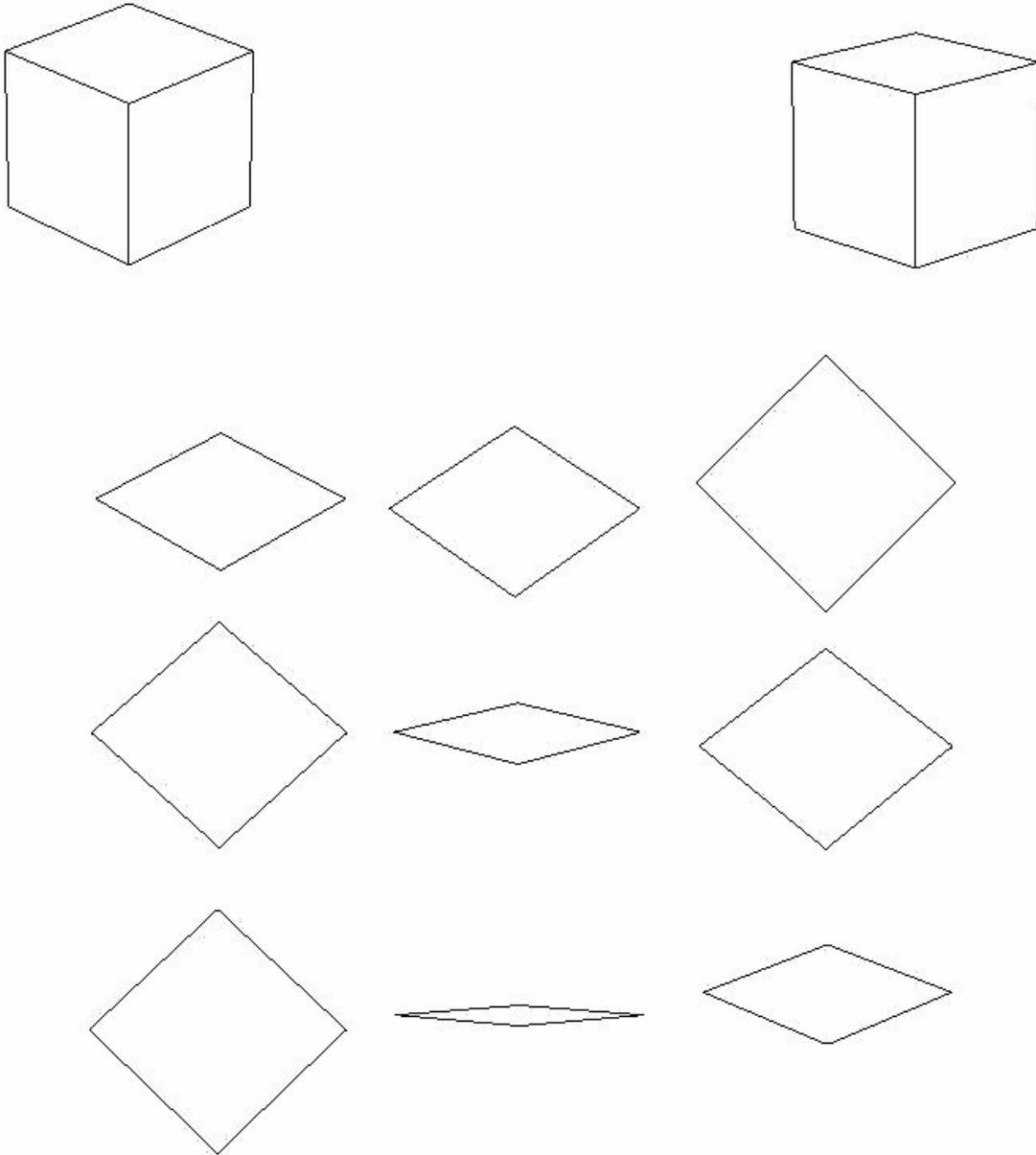


Figure 4.

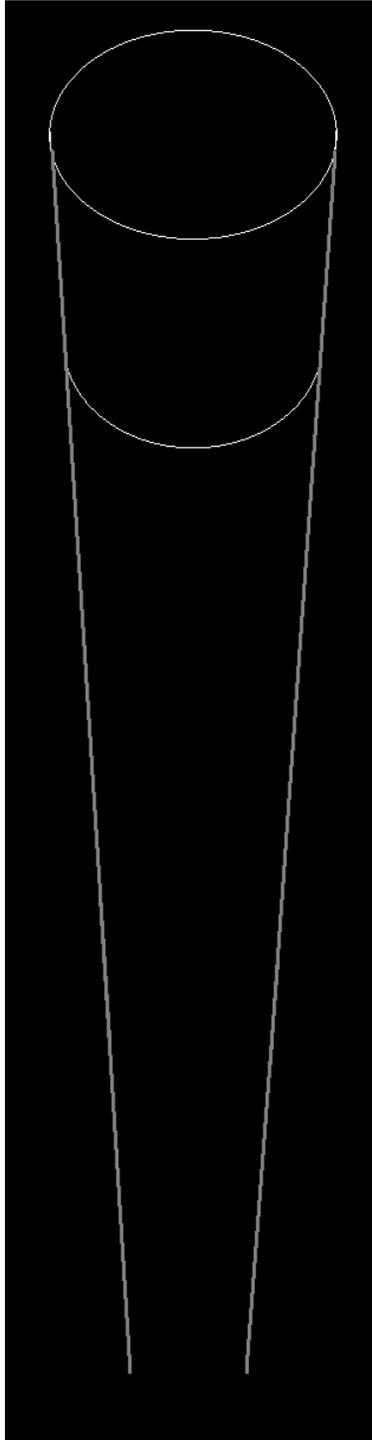


Figure 5.

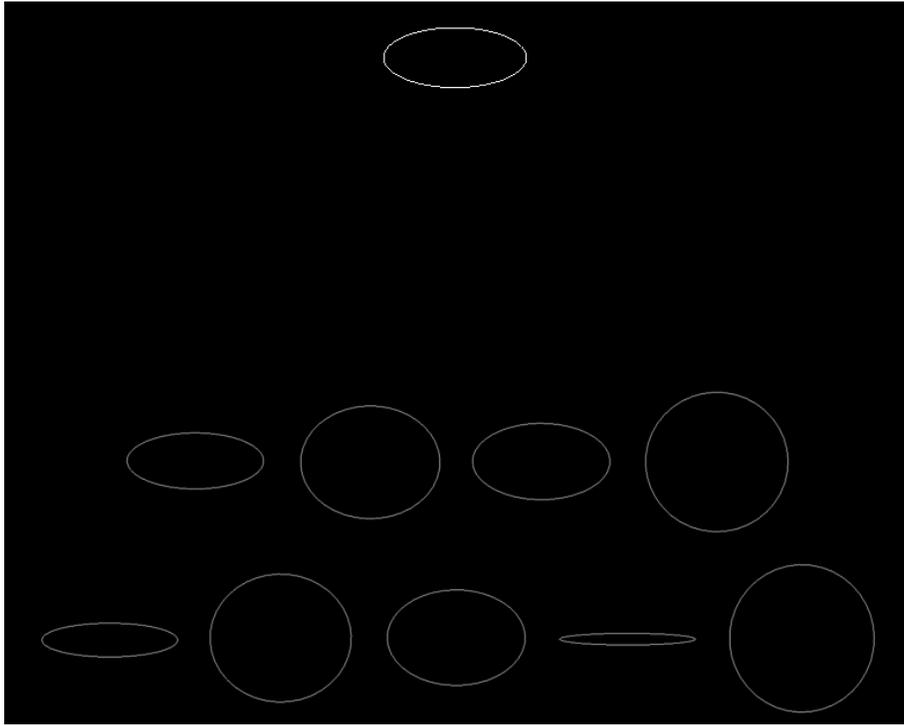


Figure 6. Mean heights of ellipse chosen at each tilt.

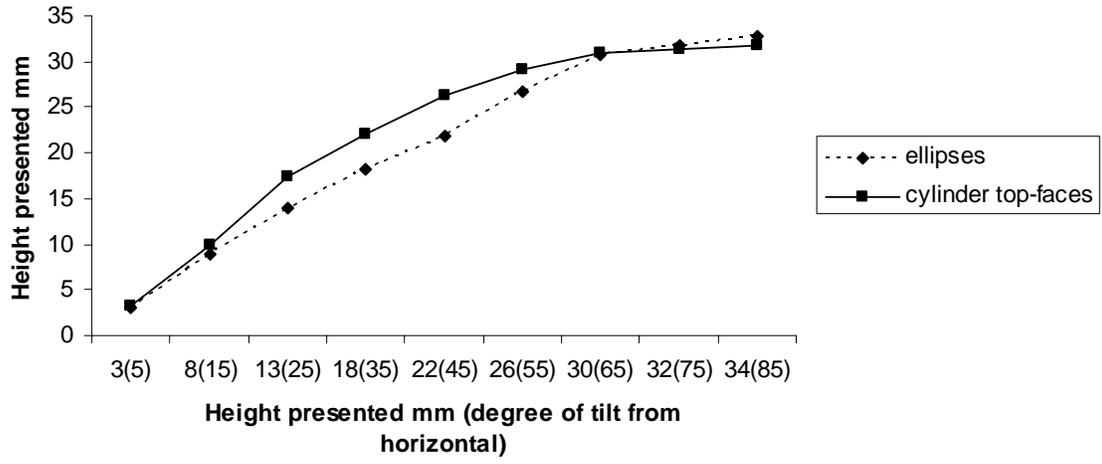


Figure 7. Mean errors at each tilt.

