CSCB63 Tutorial 10 — Expected Running Time, Stats, etc.

Recall the *indicator random variable*:

$$X_i = \begin{cases} 1 & \text{if an event } i \text{ occurs} \\ 0 & \text{if it does not} \end{cases}$$

One fact about indicator random variables that we use a lot is that

$$E[X] = Pr(X = 1)$$

Bernoulli distribution is the discrete probability distribution of a random variable which takes the value 1 with probability p and the value 0 with probability $q = 1 - p$. The expected value of a Bernoulli random variable X is $Pr(X = 1) = p$.

If we have n independent Bernoulli random variables, then the sum is the *binomial* distribution:

$$Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

is the probability of having k successes out of n total trials.

The *geometric* distribution gives the probability that the first occurrence of success requires k independent trials, each with success probability p. If the probability of success on each trial is p, then the probability that the k^{th} trial is the first success is

$$Pr(X = k) = (1 - p)^{k-1} p$$

And the expected number of trials until the first success is

$$E(X) = 1/p$$

- Let's say we toss a fair coin that has a probability p of landing "heads" each time. Then the expected number of heads in n tosses is:
- (from CLRS) Suppose n customers leave their respective hats at coat check. Later the customers receive their hats back but randomly (uniformly) permuted. What is the expected number of customers who are lucky enough to get back their own hats?
- Consider the following piece of code. What is the expected final value of the variable x?

```
x := 0
do n times:
  if random(0,1) <= 3/4:
    x := x + 5
  else:
    x := x - 1
```

- (from CLRS) Suppose we repeatedly toss a ball into one of b bins randomly. What is the expected number of tosses until every bin is non-empty?
- This one is more difficult. Consider the following piece of code:

```
c := 1
t := 0
while c <= n:
  t := t + 1
  if random(1, 2^c) = 1:
    c := c + 1
```

What is the expected final value of t? (Note that this amounts to the expected running time.)