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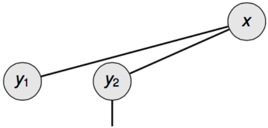

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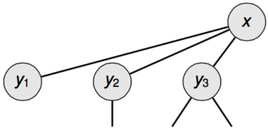

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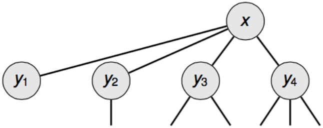

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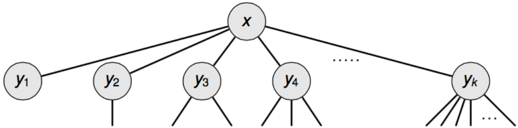

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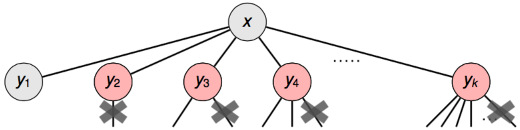

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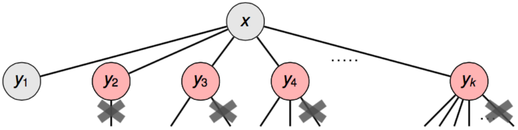

Observation. $\forall i, 1 \leq i \leq k: \; d_i \geq i-2$

Bounding the number of nodes

Let's determine the *minimum* number of nodes $N(k)$ possible in a *tree* with root of degree k.

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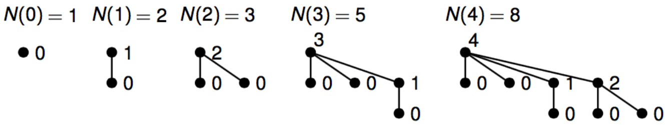

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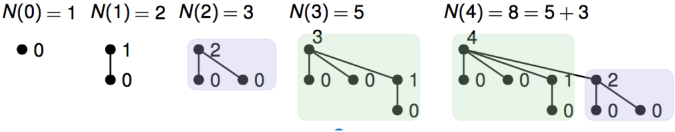

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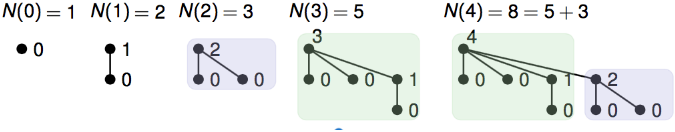

Observation. $N(k) = N(k-1) + N(k-2) = F(k+2)$?

where $F(k+2)$ is the $k+2^{nd}$ Fibonacci number.

N(k) = F(k+2)?

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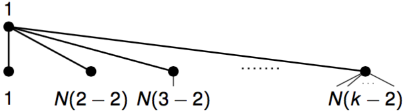

$$
\begin{aligned}
N(k) &= 1 + 1 + N(2-2) + N(3-2) + \cdots + N(k-2) \\
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&= \\
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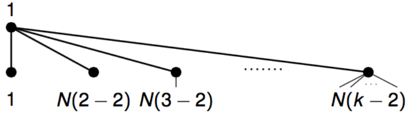

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N(k) &= 1 + 1 + N(2-2) + N(3-2) + \cdots + N(k-2) \\
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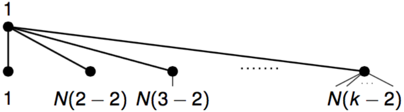
$N(k) =$ 

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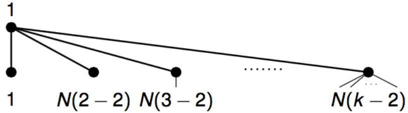

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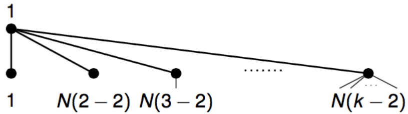

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$O(d(n)) \in O(\log n)$

Lemma. For all integers $k \geq 0$, $F(k+2) \geq \varphi^k$ where $\varphi = \frac{(1+\sqrt{5})}{2} = 1.61803\ldots$.

Q. What is φ?

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We can prove this by *induction* on k.

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$\Rightarrow$ $\log_\varphi n \geq k$ where k is...$d(n)$.

Therefore,

`Extract_Min` amortized cost of $O(d(n))$ is really $O(\log n)$.