CSCB63 – Design and Analysis of Data Structures

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[1]based on notes by Anna Bretscher and Albert Lai

priority queue

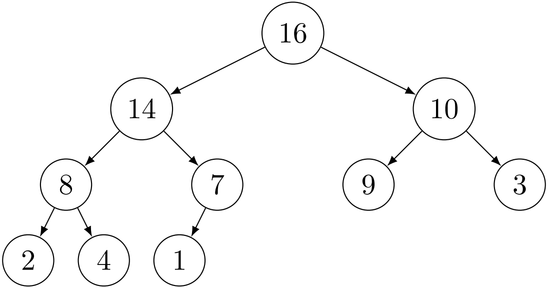
Collection of priority-job pairs; priorities must be comparable.

- `insert(`p`, `j`)`: insert job j with priority p
- `max()`: return job with max priority
- `extract-max()`: remove and return job with max priority

heap

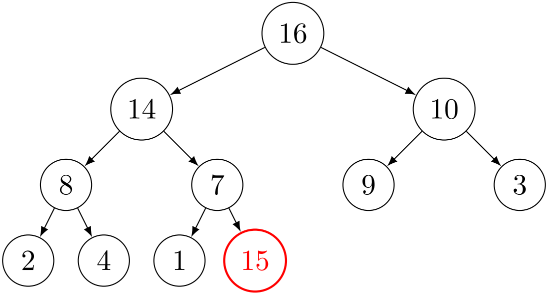
A heap is one way to store a priority queue. A heap is:

- a binary tree
- "nearly complete": every level i has 2^i nodes, except the bottom level; the bottom nodes flush to the left
- at each node n: $priority(n) \geq priority(n.left)$ and $priority(n) \geq priority(n.right)$



heap insert: example

Insert job with priority 15.



- $\surd$ The tree is still "nearly-complete". But:
- ! Order of priorities bad. Fix: swap with parent.

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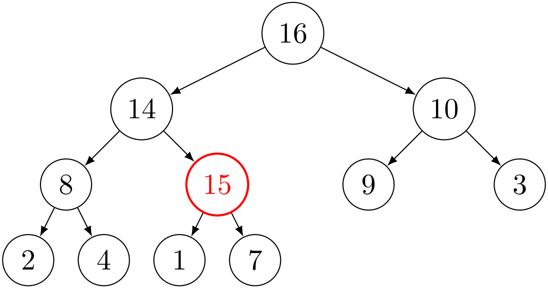

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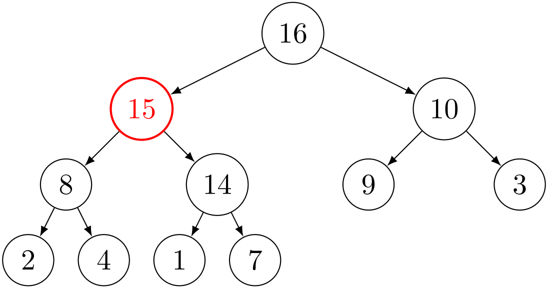
heap insert: example

Insert job with priority 15.



- $\surd$ The tree is still "nearly-complete". But:
- $\surd$ Order of priorities good.

heap insert: algorithm

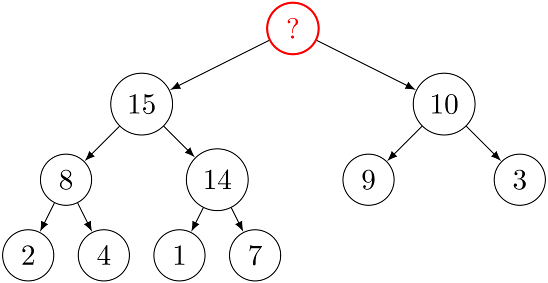
`insert(p, j)`:

1. `v := new node(p, j)`
2. insert `v` at bottom level, leftmost free place
   (keep the tree "nearly-complete")
3. while `v` has parent `p` with `p.priority < v.priority`:
   - swap `v.priority` and `p.priority`
   - swap `v.job` and `p.job`
   - `v := parent(v)`

Worst case time: $\Theta(\mathit{height})$

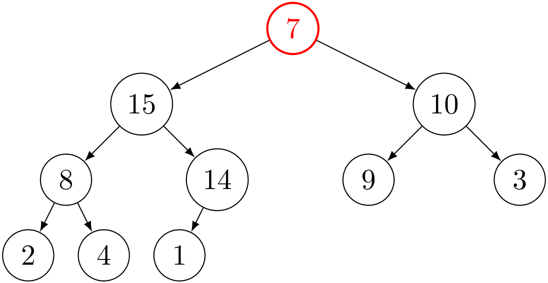
Later we will see why $\mathit{height} = \lfloor \log n \rfloor + 1$. Therefore worse case time $\Theta(\log n)$.

heap extract-max: example



new root?

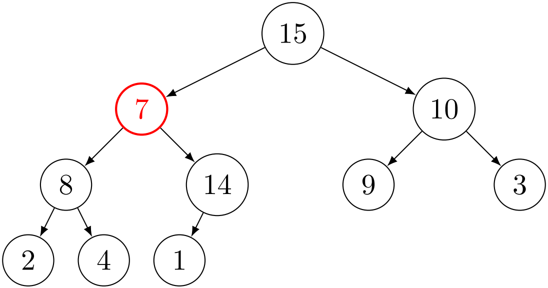
heap extract-max: example



replace by the bottom level, rightmost item.

- $\surd$ The tree is still “nearly-complete”.
- ! Order of priorities bad. Fix: swap with the larger child. (Why not the smaller child?)

heap extract-max: example



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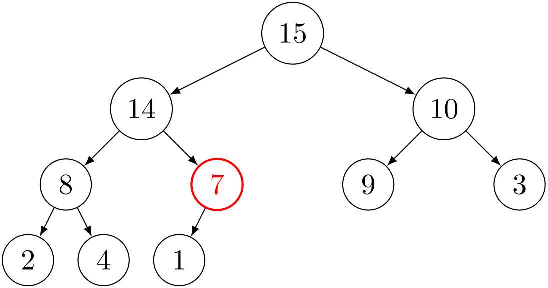


replace by the bottom level, rightmost item.

- $\surd$ The tree is still "nearly-complete".
- $\surd$ Order of priorities good.

heap extract-max: algorithm

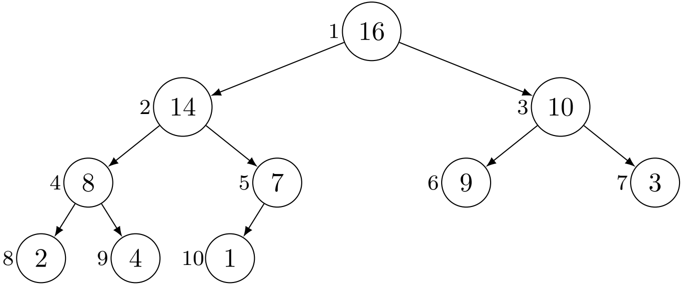
`extract-max()`:

1. `max_p, max_j = root.priority, root.job`
2. move `(priority, job)` from last (bottom, rightmost) node into root
3. remove last node
4. `v := root`
5. while `v` has child `c` with `c.priority > v.priority`:
   - `c :=` child of `v` with largest priority
   - swap `v.priority` and `c.priority`
   - swap `v.job` and `c.job`
   - `v := c`
6. return `max_p, max_j`

Worst case time: $\Theta(\textit{height})$

Later we will see why $\textit{height} = \lfloor \log n \rfloor + 1$. Therefore worse case time $\Theta(\log n)$.

heap in array/vector



	16	14	10	8	7	9	3	2	4	1	
0	1	2	3	4	5	6	7	8	9	10	11

heap in array/vector

	16	14	10	8	7	9	3	2	4	1	
0	1	2	3	4	5	6	7	8	9	10	11

Easy:

- where to insert/remove? simply at the end
- saves space: no pointers to store

Where are children/parents?

- left child of node at index i: at index $2 \times i$
- right child of node at index i: at index $2 \times i + 1$
- parent of index node at i: at index $\lfloor i/2 \rfloor$

Downside?

heap: height

Let n be the number of nodes, h be the height.

- largest n: bottom level is full
  - $n = 2^h - 1$
- smallest n: only 1 node at bottom level
  - $h - 1$ levels are full
  - $n = (2^{h-1} - 1) + 1$

$$
\begin{aligned}
(2^{h-1} - 1) + 1 \leq & \quad n & \leq 2^h - 1 \\
2^{h-1} \leq & \quad n & < 2^h \\
h - 1 \leq & \quad \log_2 n & < h \\
h \leq & \quad (\log_2 n) + 1 & < h + 1 \\
h = & \quad \lfloor \log_2 n \rfloor + 1 &
\end{aligned}
$$