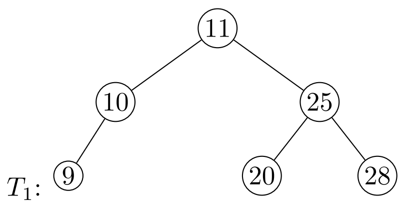
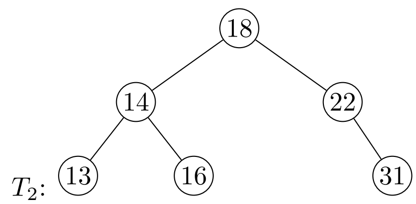
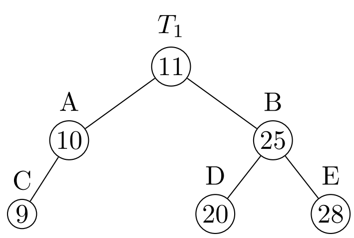
CSCB63 Tutorial 3 — Union of AVL trees

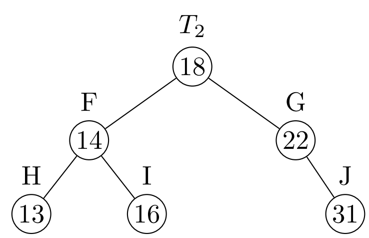
Show every step of performing the union of the two trees below:



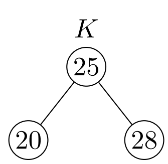


Let's label each node so we can refer to the subtrees rooted at various nodes as we work through the union process.

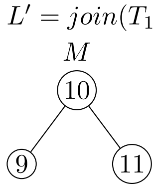




1. Divide: $(L_0, R_0) = split(T_1, 18)$
   (a) $18 > 11$, therefore we perform $(L_1, R_1) = split(B, 18)$
      i. $18 < 25$, therefore we perform $(L_2, R_2) = split(D, 18)$
         A. $18 < 20$, therefore we perform $(L_3, R_3) = split(nil, 18)$
            We get $(L_3, R_3) = (nil, nil)$ and so
            $R'_3 = join(R_3, 20, D.right) = join(nil, 20, nil) = 20$
            This returns $(L_3, R'_3) = (nil, 20)$, and so $(L_2, R_2) = split(D, 18) = (nil, 20)$
      ii. We now compute $R'_2 = join(R_2, 25, B.right) = join(20, 25, 28) = K$



      This returns $(L_2, R'_2) = (nil, K)$, and so $(L_1, R_1) = split(B, 18) = (nil, K)$
   (b) We get $(L_1, R_1) = (nil, K)$ and so
      $L' = join(T_1.left, 11, L_1) = join(A, 11, nil) = M$:



   And so we have $(L_0, R_0) = split(T_1, 18) = (M, K)$

2. Conquer:

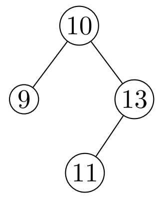
(a) Perform $union(L_0, F) = union(M, F)$

i. Divide: $(L_1, R_1) = split(M, 14) = (M, nil)$ by following a process similar to what we just did for $split(B, 18)$.

ii. Conquer:

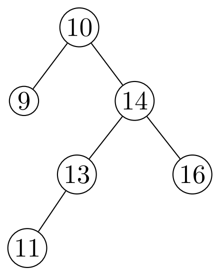
A. Perform $union(M, 13) = \ldots = join(M, 13, nil) = N$:

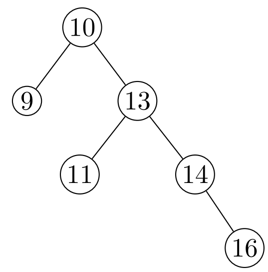
N



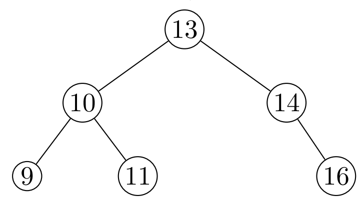
B. Perform $union(nil, 16) = 16$

iii. Perform $join(N, 14, 16)$ — need a double rotation:





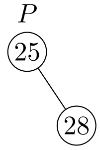
Q



(b) Perform $union(R_0, G) = union(K, G)$

i. Divide: $split(K, 22) = (20, P)$:

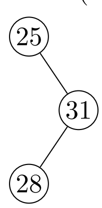
P

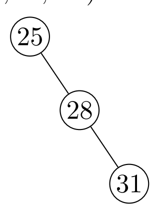


ii. Conquer:

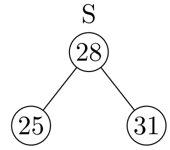
A. $union(20, nil) = 20$

B. $union(P, 31) = join(P, 31, nil)$ — need a double rotation:



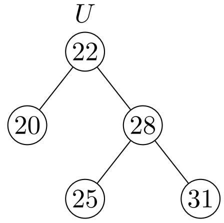


S

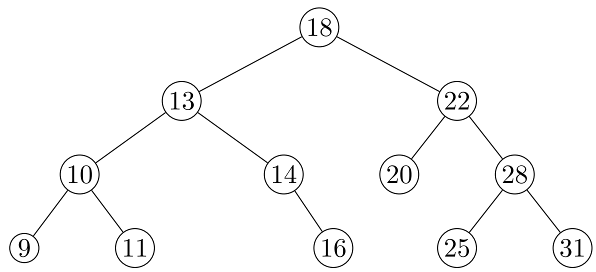


iii. Perform $join(20, 22, S) = U$:

U



3. Finally, $join(Q, 18, U)$ to get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