CSCB63 Tutorial 1 — Asymptotic Bounds

1 common time complexity asymptotic upper bounds

Consider the following 2D table with column headers and row headers both being: $\ln(n)$, $\lg(n)$, $\lg(n^2)$, $(\lg n)^2$, n, $n\lg(n)$, 2^n, 2^{3n}. Recall that lg is log base 2 and ln is natural logarithm (base e).

In each cell (row, col) fill in `Y` if *row*'s function is Big-Oh of *col*'s function.

	$\ln(n)$	$\lg(n)$	$\lg(n^2)$	$(\lg(n))^2$	n	$n\lg(n)$	2^n	2^{3n}
$\ln(n)$								
$\lg(n)$								
$\lg(n^2)$								
$(\lg(n))^2$								
n								
$n\lg(n)$								
2^n								
2^{3n}								

As an exercise, prove transitivity of Big-Oh:

$$f \in O(g) \wedge g \in O(h) \Rightarrow f \in O(h)$$

Note that even though $3n \in O(n)$, we cannot "exponentiate both sides" to infer $2^{3n} \in O(2^n)$.

2 formal proofs of asymptotic upper bounds

We will show proofs for two entries in our Big-Oh table. We will use the Big-Oh definition directly.

1. Prove $n \in O(n \lg n)$.

2. Prove $n\lg(n) \notin O(n)$.

3 common time complexity asymptotic tight bounds

Now develop a similar table for Big-Theta: in each cell (row, col) fill in `Y` if *row*'s function is Big-Theta of *col*'s function.

	$\ln(n)$	$\lg(n)$	$\lg(n^2)$	$(\lg(n))^2$	n	$n\lg(n)$	2^n	2^{3n}
$\ln(n)$								
$\lg(n)$								
$\lg(n^2)$								
$(\lg(n))^2$								
n								
$n\lg(n)$								
2^n								
2^{3n}								

4 optional exercises

1. Prove $6n^5 + n^2 - n^3 \in \Theta(n^5)$ using the definition of Big-Theta.
2. Prove $3n^2 - 4n \in \Omega(n^2)$ using the definition of Big-Omega.