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The Perfection of Bodies: Aristotle's *De Caelo* I.1

Abstract: In this paper we give a detailed reconstruction of the first chapter of *De Caelo* I.1. Aristotle attempts to prove there that bodies are complete and perfect in virtue of being extended in three dimensions. We offer an analysis of this argument and argue that it gives important insight into the role the notion of body plays in physical science. Contrary to other interpretations, we argue that it is an argument about physical, as opposed to mathematical, bodies and that the perfection and completeness of bodies is due to their nature. Moreover, Aristotle heavily relies in his proof on the premise that the number three implies perfection, a view he ascribes to the Pythagoreans. We review the possible sources of this view, as well as its role in Aristotle's argumentative strategy.

Keywords: Aristotle, body, physical science, Pythagoreans.

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Nearly sixty years ago Olof Gigon remarked that what was missing from discussions of Aristotle's philosophy were detailed studies of how Aristotle runs an argument in the course of a single chapter.[1] As a first attempt at remedying the situation, Gigon himself provided a detailed analysis of the first chapter of *De*

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1 Gigon (1952), p. 114: 'Was der Aristotelesforschung heute noch am meisten zu fehlen scheint, sind Interpretationen, also Untersuchungen, die dem Kontinuum eines bestimmten Textes nachgehen und den Sinn jedes einzelnen Satzes für sich und in seiner Umgebung zu verstehen suchen.'

Caelo. Since then, we have seen a proliferation of studies methodically focusing on single Aristotelian chapters and shorter stretches of text.

The aim of our paper is to return to the first chapter of *De Caelo* and to offer a new detailed analysis of the structure of the argument set out in this apparently patchy text. The chapter, we wish to claim, raises intricate questions that relate to issues that are at the core of Aristotle's philosophy of nature and metaphysics. The notion of body as Aristotle develops it in this chapter is not only the key for understanding the structure of the chapter as a whole and the way in which the main topic of the treatise, i.e., the cosmos, is introduced. Understanding this notion can also contribute to unlocking deep puzzles concerning the relation between physical and mathematical bodies or the relation between the nature of physical bodies and their dimensions as they are discussed in other works, such as the *Physics* or the *Metaphysics*. Moreover, it may also show that the notion of body is indeed crucial to Aristotle's project of physical science as a whole. The first chapter of *De Caelo* is, we believe, part of an explicit analysis of the notion of body. As it stands, we do not find such an explicit analysis in the *Physics* nor does Aristotle announce body as a topic to be discussed in the *Physics* as he does with the notions of place, time, and motion.[2] Thus, an analysis of *De Caelo* I.1 may show that Aristotle is not oblivious of the importance of the notion of body for physical science. Therefore, though our discussion is restricted to the *De Caelo*, the ultimate aim of our paper is to make a contribution to discussions that extend beyond the confines of this work.

Here then is an overview of the chapter and our argument: The chapter consists of three parts that present three distinct but related topics. In a brief proem (268^{a}1–6) Aristotle states the subject matter of physical science. He remarks that physical science deals with bodies, their affections and principles. Therefore, it is the task of the physicist to study bodies. Since animals and other physical objects have extended bodies, this may seem uncontroversial. However, as the second part, which takes up the main bulk of the chapter (268^{a}6–268^{b}5), shows, Aristotle has a more ambitious project here. For he turns to a general and abstract analysis of the notion of body, focusing on the perfection or completeness of bodies, before claiming in a relatively brief third section (286^{b}5–10) that the perfection or completeness of the bodies that make up the cosmos is qualified, whereas the cosmos – and the cosmos alone – is perfect and complete (τέλειον) in an unre-

2 It is an interesting question why he does not do so, but a conjecture is that from *Physics* III onwards Aristotle is chiefly concerned with motion and its preconditions and thus he discusses the notion of body only where it crops up in the analysis of motion.

stricted sense. In the centre of the chapter Aristotle thus presents an analysis of physical bodies *as such*.

This central part of the chapter uses a baffling mixture of arguments and the emphasis of our paper will be on this part. Aristotle first defines body via the notion of three-dimensional extension (268^{a}6–10). He then uses this characterisation of body to argue for the claim that completeness and perfection are not extra attributes conferred upon bodies. Bodies are perfect insofar as they are bodies. In other words, bodyhood entails perfection (268^{a}20–^{b}5). We shall argue that the argument concerns specifically physical bodies, as opposed to geometrical solids, and support our claim with reference to parallel passages.

Moreover, Aristotle relies in his arguments on allegedly Pythagorean doctrines, Greek cult practices, and linguistic data (268^{a}10–20). Given that in other texts Aristotle is usually highly critical towards Pythagoreans and their theory of numbers, this aspect of the text is obviously in need of explanation.

It is in the final part (268^{b}5–10), just as brief as the proem, that Aristotle reaches the proper theme of the treatise, i.e., the cosmos.[3] Thus, in view of the announced subject matter of the treatise, the introductory chapter can be read as gradually leading up to this climax stating the supreme perfection of the body which is the cosmos.[4]

Up to this final part Aristotle has given no indication that the cosmos differs from other bodies with respect to their previously stated completeness and perfection or, for that matter, that the cosmos is one of the bodies. One might even have got the impression that the cosmos was not in the scope of the discussion at all. At this point, however, Aristotle states that while the completeness of particular bodies[5] is qualified by the reciprocal delimiting effect they have on one another, the completeness of the cosmos is unrestricted and hence of a higher order. This is a surprising claim because it does not seem to be the case that the boundaries of a table are determined by the surrounding air. In our analysis of this passage we will argue that with the expression ‘bodies in the form of parts’ (268^{b}5) Aristotle is referring to the elements rather than individual physical objects, like trees, animals, or tables.

3 It is worth noting in this context that Aristotle waits until chapter 9 of the first book before he distinguishes several meanings of the word cosmos (*ouranos*).

4 We tend to think that, in the Aristotelian framework, one can talk about a single body only when this body is the body of a single substance – but it is not unproblematic to view the cosmos as a single bodily substance. The cosmos encompasses several bodily substances (e.g., the simple bodies and living beings) and it is unclear whether a substance can be composed of several other substances.

5 ‘Particular bodies’ is our shorthand for what Aristotle calls ‘bodies in the form of parts’ (268^{b}5).

The proem: body as the subject matter of physical science (*Cael*. I.1 268^{a}1–6)

The first part concerning the proper and primary objects of physical science compares in a complex and non-obvious way to other characterizations of the subject matter of physical science we find elsewhere in the corpus. We shall argue that Aristotle's programmatic statement is indeed in line with his characterization of the objects of physical science elsewhere, but that the formulation in *Cael*. I.1 lays a special emphasis on the notion of body, thereby paving the way for the introduction of the cosmos as the most perfect and complete body.

Aristotle begins the chapter and therefore the whole treatise with a general statement about the subject matter of physical science. The inquiry into nature is, he says, 'for the most part about bodies and magnitudes, their affections, the movements and further the principles of such a substance'.[6]

This immediately raises the question concerning the role of this sentence. We think that it has two functions. The first is to properly locate the notion of body that will be discussed subsequently. This means that physical bodies are the focus of the following analysis. Second and closely related, it shows that the notion of body that is presented in the first chapter of *De Caelo* is of general interest to the physicist, insofar as it does not pertain exclusively to the cosmos.

Having said that, there is the prior question whether Aristotle's characterization of magnitudes and physical science is in accordance with characterizations we find elsewhere. As we know from *Metaphysics* E.1, physical science is concerned with 'such a substance that has in itself the principle of movement and rest' (Aristotle, *Metaph*. E.1 1025^{b}19–21).[7] Aristotle's standard examples for it are simple bodies (i.e. fire, earth, water, air) and things made of them, as, e.g., the heaven and its parts and likewise plants, animals and their parts.[8] Moreover, Aristotle often calls these substances 'physical bodies'.[9] Since physics studies physical substances (i.e., substances which have a principle of movement and

6 Ἡ περὶ φύσεως ἐπιστήμη σχεδὸν ἡ πλείστη φαίνεται περί τε σώματα καὶ μεγέθη καὶ τὰ τούτων οὖσα πάθη καὶ τὰς κινήσεις, ἔτι δὲ περὶ τὰς ἀρχάς, ὅσαι τῆς τοιαύτης οὐσίας εἰσίν. (Arist. *Cael*. I.1 268^{a}1–4) We understand φαίνεται at line ^{a}1 in the sense of 'it is obvious that…', since it is used in a construction with a participle.

7 See also Aristotle, *Ph*. II.1 192^{b}8–193^{a}2 (in particular lines 192^{b}8–15 and 192^{b}32–193a2) and *Ph*. III.4 202^{b}30–31.

8 Cf. e.g. Aristotle, *Metaph*. H.1 1042^{a}6–11; *Cael*. III.1 298^{a}28–b5; *PA* I.5 644^{b}22–30.

9 Cf. Aristotle, *Metaph*. Z.2 1028^{b}8–14: bodies as substances: animals, plants, their parts, physical bodies as fire, earth etc., their parts and what derives from them (as the heaven and its parts); *de An*. II.1, 412^{a}13. Cf. *Metaph*. H.1 1042^{a}6–11 physical substances: fire, earth, … simple bodies, rest in them) and all physical substances are bodies, the greater part of physical investigations have to do with bodies. And since the knowledge of bodies is intimately tied to their various attributes, they should be in the scope of physical science as well.[10] In short, we think that Aristotle reasons in the first lines of the chapter as follows: Physical science studies things that are by nature. The things that are by nature are (1) bodies and magnitudes or (2) affections of body and magnitude or (3) principles in virtue of which bodies have these specific affections. Therefore, physical science studies (1) bodies and magnitudes or (2) affections of body and magnitude or (3) principles in virtue of which bodies have these specific affections.[11]

Aristotle thus provides a general conception of the subject matter of physical science which accords with his characterization of the subject matter in other works.[12]

The reason why the proem may initially sound unfamiliar is that it concentrates particularly on the relevance of the notion of body. But given what follows in the chapter this is a sensible thing to do. For Aristotle in this way carefully introduces the main topic of the first chapter by emphasizing the importance of the notion of body for natural science. In the main part of the chapter then Aristotle analyzes the notion of body that is pertinent to physical science.

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plants, and animals and their parts, heaven and its parts; *Cael.* III.1 298[b]1–5: physical substances are bodies or together with bodies and magnitudes.

10 As Aristotle makes clear in the next line, the items he just enumerated, i.e. magnitudes, their affections and principles, are 'things constituted by nature' (cf. Aristotle, *Cael.* I.1 268[a]4–6). This addition is important because it narrows down the scope of the magnitudes in question. For even if all natural substances are bodies, it does not follow that all bodies are natural substances. Hence, the scope of physics as specified in the first sentence of *Cael.* I.1 is broader than the *Metaphysics* E.1 and *Physics* II.1 specification. It is only in the second sentence that the scope is restricted to things constituted by nature.

11 We defend our translation and interpretation of the sentence in Appendix 1.

12 Sometimes he characterizes physical science as the study of nature. Cf. Aristotle, *Ph.* III.1 200[b]12–14. That is, Aristotle speaks as if the proper subject matter of physical science is nature as a principle, rather than the things that have such a principle. We explain that by the fact that in the first four books of the *Physics* Aristotle is mainly interested in nature as a principle since a clarification of the concept of nature is the first task the physicist has to undertake. For the natural order of physical investigation, see Aristotle, *Mete.* I.1 338[a]20–339[a]9. For a discussion, see Burnyeat (2004).

A conception of body (*Cael.* I.1 268^{a}6–^{b}5)

Cael. I.1 268a6–10: Body as three-dimensional entity

In the following lines Aristotle jumps right into the main discussion by giving a definition of body:

> Hence continuous is that which is divisible into ever divisible parts, body is that which is divisible in every way. Of magnitude, that which is extended in one dimension is a line, that which is extended in two is a surface and that which is extended in three dimensions is a body. There is no other magnitude beyond these, since the three is all and the thrice is in every way. (*Cael.* I.1 268^{a}6–10)

This definition may come as a surprise. It might give the impression that Aristotle switches his topic from a very general conception of physical science to a definition of abstract mathematical entities.[13] This reading may be encouraged by the not implausible assumption that the first chapter of *De Caelo* is actually a patchwork of different texts, originally formulated in different contexts. The transition is no doubt somewhat abrupt, but it is not completely out of line. Indeed, as should be clear from our analysis of the proem, we believe that Aristotle has prepared the ground for a closer analysis of body, thereby establishing the framework for the ensuing discussion. After having introduced body as the main subject of physical science, Aristotle offers in the next sentences an account of body (note the particle οὖν). Aristotle is therefore concerned with an analysis of natural bodies which are the subject matter of physical science.

Yet, it is noteworthy that the ensuing definition of body focuses exclusively on apparently mathematical properties, namely, three-dimensionality and divisibility. This has puzzled commentators and may lead to the view that Aristotle wants to leave open whether the magnitudes are physical or mathematical.[14] Although it is true that the notions of three-dimensionality and divisibility do

13 As an example of such a view see Wildberg (1988), p. 20.

14 Cf. Falcon (2005), p. 38: 'In this context, Aristotle does not intend to provide the best possible definition of body; that is, a definition that among other things may enable him to distinguish a body from a geometrical solid'. See also p. 48: 'The fact that the *Timaeus* is a polemical target of the *DC* explains why Aristotle begins this treatise with a minimal notion of body; a notion that, among other things, does not distinguish bodies from geometrical solids'. Wildberg (1988), p. 28: 'At the same time, the methodology, which becomes apparent in the peculiar absence of a clear distinction between the mathematical solid and the physical body, requires an explanation within the framework of Aristotle's natural philosophy'.

not distinguish geometrical solids from physical bodies, it does not follow that Aristotle *deliberately* leaves open the question whether he speaks about physical bodies or geometrical solids. It only follows that Aristotle is interested in an aspect of physical bodies that they share with geometrical solids. This datum is indeed in need of explanation, but we think it is misleading to say that Aristotle does not distinguish between physical bodies and geometrical solids. It is especially misleading, because it suggests that there are geometrical solids independently of physical bodies. The question whether one studies a geometrical solid or rather a physical body is for Aristotle not a question of ontology, but a question regarding the form of inquiry. For Aristotle there are no geometrical solids over and above physical bodies. A geometrical solid is a physical body considered in a certain way. The geometer studies, according to Aristotle, physical bodies and their attributes, but not insofar as they are physical or insofar as the attributes belong to physical bodies.[15]

It is therefore wrong to deduce from the fact that Aristotle apparently defines magnitudes with regard to their extension that he is engaged in a mathematical project. Rather, what he does is singling out an aspect of sensible magnitudes that the physicist and the mathematician study.[16]

If we consider the question in this way, it is clear that the first chapter of *De Caelo* is concerned with physical bodies. Already in the proem Aristotle makes clear that the subject matter of the treatise is physical science. Moreover, Aristotle turns in the second chapter of the first book to the other defining feature of physical bodies, viz., their having a principle of motion and rest. Thus, by the second chapter a complete description of the main subject matter of physical science is reached.

Additionally, one should note that physical bodies have strong metaphysical and epistemological priority for Aristotle. So it is also plausible that ‘body’, without qualification, will refer to physical bodies that can, if certain conditions obtain, be viewed as mathematical. Thus, without any further context, the default reference for any definition of body will be physical bodies. This is especially so when the context is the proper object of physical science.

Yet, one may wonder why Aristotle singles out this particular aspect of bodies. Why does he begin his remarks about magnitudes and especially bodies not with some comments about their nature, but rather about their extension? After all,

15 For some suggestions that Aristotle had developed his mathematical views by the time of the *De Caelo* see Appendix 3.

16 As remarked above, it is clear from *Physics* II.2 that Aristotle believes that the physicist studies lines, planes, and bodies, too.

it is true that what defines physical bodies *qua* physical is their having a nature. So shouldn't we expect that Aristotle would remark on the nature of the bodies before considering other aspects of them? We shall argue that any such expectation underestimates the force of the argument that Aristotle presents in the main part of the chapter: Aristotle's 'proof' that objects cannot be extended in more than three dimensions and that among magnitudes body is complete (τέλειον).[17]

For one of the main aims of the chapter, as we shall show, is precisely to demonstrate that perfection directly follows from bodyhood. The cosmos is perfect simply in virtue of being the maximal body, irrespective of other properties – geometrical or other – it may have. Being extended in three dimensions is not any old property among others that physical bodies have, but which is of less interest than their nature to the physicist. Being extended in three dimensions is of the utmost importance because it – and it alone – explains why bodies are perfect. This is an important insight into the nature of physical bodies and it is not mainly a mathematical result.[18]

If this claim is correct, the question arises how the nature of physical substances and the fact that they are three-dimensional are connected. Though Aristotle does not explicitly address the question, we believe that he hints at some answer in lines 268^{b}1–5.

Before turning to that we will offer an analysis of the central section of the chapter. In a first step, Aristotle argues that being extended in three dimensions and being divisible in three ways is being extended in *all* dimensions and being divisible in *all* ways (cf. *Cael.* I.1 268^{a}10–20). In a second step, he argues – quite briefly – that body alone is complete (*Cael.* I.1 268^{a}20–28).

17 We shall make some remarks on the translation of the term τέλειον below.

18 It is plausible to assume that for Aristotle the priority of three-dimensional items which are complete over less-dimensional items is not applicable within mathematics. The mathematical sciences are ordered according to logical priority. Thus, plane geometry is prior to solid geometry. The science solely concerned with two-dimensional surfaces is prior to the science concerned with solids. Ontologically, however, the priorities run the other way round: 'Again, the modes of generation show that we are right. First length is generated, then breadth, lastly depth, and it is complete. If, then, that which is posterior in generation is prior in substance, the body should be prior to plane and length. It is more complete and whole in the following way also – it becomes animate. How, on the other hand, could there be an animate line or a plane? The supposition passes the power of our senses' (Aristotle, *Metaph.* M.2 1077^{a}24–31, transl. Annas 1976, modified). This passage is obviously close to the first chapter of *De Caelo*. In both cases we find a claim about the priority of bodies that is connected to the substantiality of natural bodies and an (admittedly puzzling) argument about the generation of bodies from lines and planes. Moreover, both arguments rely on the principle that what is later in generation is prior in substance. Cf. below.

Cael. I.1 268ᵃ10–20: Three is all

Let us first have a closer look at lines 268ᵃ6–10 again:

> Hence continuous is that which is divisible into ever divisible parts, body is that which is divisible in every way. Of magnitude, that which is extended in one dimension is a line, that which is extended in two is a surface and that which is extended in three dimensions is a body. There is no other magnitude beyond these, since the three is all and the thrice is in every way. (*Cael.* I.1 268ᵃ6–10)

Aristotle introduces the notions of divisibility and continuity and apparently defines 'body' as that which is divisible in three ways and continuous in three dimensions. Analogously, a line is divisible in one way and one dimension and a surface in two dimensions. He ends with the claim that there cannot be another, further magnitude besides these because divisibility in three ways and continuity in three dimensions is divisibility in all ways and continuity in every dimension. Hence, only these three types of magnitude can exist. In the subsequent lines, 268ᵃ10–20, Aristotle vindicates this claim by connecting 'three' with 'all'.

Though in terms of argument structure that is without doubt a sensible thing to do, the language and style of the following paragraph has led some commentators, such as Gigon,[19] to believe that Aristotle interpolated it from a different text, possibly an exoteric dialogue. This might very well be so, yet we still have to be aware of the fact that the central claims of the chapter rest precisely on these arguments. In order to demonstrate that three-dimensional objects are *teleia*, i.e. perfect / complete,[20] due to the mere fact that they are extended in exactly three dimensions, Aristotle has to show that three and thrice generally entail perfection and completeness, so that all instantiations of three and thrice are complete and perfect. His main line of argument relies thereby on the idea that the three is generally an 'all' and a sort of totality such that there cannot be more than three dimensions because being extended in three dimensions is being extended in *all* dimensions. And being extended in *all* dimensions conveys – in a sense we will explain in a moment – perfection and completeness to bodies. These arguments are thus not simply there to add some rhetorical flourish to the text or to provide some interesting cultic, ethnographic, and linguistic analogies and parallels, but are supposed to carry pretty much all the weight of the principal tenet of the chapter.

19 Cf. Gigon (1952), p. 119.

20 We will discuss the meaning of *teleion* and the connection between completeness and perfection in the next section.

Yet, no matter how essential the connection between all and three is for Aristotle's overall argument, it is not at all obvious how one could demonstrate this connection scientifically or substantiate it by properly philosophical reasoning. Indeed, Aristotle apparently cannot do better but appeal to an alleged Pythagorean doctrine, to a set of cult practices, and to Greek linguistic usage. By all appearances, these *phainomena* should convey, first, the expert view on the question, and, then, the opinion of most people, or at least most Greeks.

Let us take Aristotle's three pieces of evidence in reverse order. Given his conventionalist views on language, Aristotle cannot of course immediately derive from linguistic data lessons about the way things are. Linguistic usage can, on the other hand, reflect (early) people's views about things, at least to some extent and in so far as people established linguistic conventions on the basis of their shared views and intuitions about the world. And people could have correct intuitions about the way things are – could 'let nature guide them' – and henceforth their linguistic innovations can indeed reflect the nature of reality. This is also why, on rare occasions, Aristotle includes etymology among the *endoxa*. One notable example, the derivation of *aithêr* from *aei thein*, 'always running', occurs a few pages later in *De Caelo* I.3 270^{b}23.

It remains nevertheless true that Aristotle's specific linguistic argument can hardly serve his present purposes. For the linguistic datum that the Greeks employ the term 'all' to groups of things or people when there is at least three members of the group cannot show that members of groups with four or five elements cannot collectively be called 'all'. And what Aristotle is supposed to establish is precisely that we should stop at three dimensions, because three – and not four or five, or more – is all.

The reference to the prominence of three, or rather of three times, in cult practices operates with the same underlying assumption. Rituals are based on convention, but these conventions express commonly shared views and intuitions that can grasp some essentially important feature of the world. As Aristotle says, we have extracted the number three from nature as a *nomos* of it, and employed it in our own *nomoi* – especially in those *nomoi* that concern the worship of gods. Although Aristotle does not specify which rituals he has in mind, numerous cult actions had to be repeated three times in Greek religious practice (just as in the religious practices of numerous other peoples).[21] In the present context, the most notable might be that symposiasts ought to have made the third and final libation to Zeus *Sôtêr* also called Zeus *Teleios*. Interestingly, ancient sources on this custom often explain it with reference to the completeness or perfection (*teleios*) of the

21 For an inventory of such practices see Usener (1903).

number three.[22] It is remarkable that the practice, and Zeus' epithet *Teleios*, is explained by several scholiasts exactly in terms of the triad of beginning, middle, and end.[23] Even more interesting, these texts do not say that Zeus is *Teleios* in so far as he holds beginning, middle, and end, but in so far as the number three is *teleios* because it has the beginning, middle, and end. The completeness of Zeus is thus explained by the ancient commentators from the completeness of number three, in so far as the number three possesses beginning, middle, and end.[24]

This is exactly, or almost exactly, the view Aristotle attributes to the Pythagoreans, with the only difference that in their case it is the trio of beginning, middle, and end, that has the number three, and not *vice versa*:

> For, just as the Pythagoreans say, the whole and all things are delimited (*or* defined, *hôristai*) by the three; for end, middle, and beginning have the number of the whole, which is that of the triad. Wherefore, we use this number also in the worship of the gods, taking it from nature, as a law of it. (*Cael.* I.1 268ᵃ10–15)

The patent correspondence between the Aristotelian text and the evidence mentioned above invites two alternative interpretations. We cannot exclude the possibility that the scholiasts' explanations go back to Aristotle's formulation as we have it in the *De Caelo*. Alternatively, the connection between the completeness of the number three on account of its possessing (or being possessed by) the trio of beginning, middle, and end, and the relevant cult practices involving Zeus *Teleios* originally formed one single argument, Pythagorean or otherwise, and thus the juxtaposition of them is not due to Aristotle. At this point, we cannot decide the question either way.

Be that as it may, in the final account, this is Aristotle's principle piece of evidence for the connection between three and all. Yet, the view, as well as the attribution to the Pythagoreans, is problematic in a number of respects. First of

22 Cf. Pollux, *Onomasticon* 6.15: κρατῆρες δ' ὁ μὲν πρῶτος Διὸς Ὀλυμπίου καὶ Ὀλυμπίων θεῶν, ὁ δὲ δεύτερος ἡρώων, ὁ δὲ τρίτος Διὸς σωτῆρος τελείου, ὅτι καὶ τὰ τρία πρῶτος τέλειος ἀριθμός. For a full documentation, see Cook (1925), pp. 1123–4.

23 Cf. e.g. Schol. Pind. *Isthm.* 6.10a.18–20: ἔλεγον δὲ αὐτὸν [sc. Zeus Sôtêr] καὶ τέλειον διὰ τὸ τέλειον εἶναι τὸν τρίτον ἀριθμὸν ἀρχὴν ἔχοντα καὶ μέσον καὶ τέλος; Schol. Plat. *Charm.* 167A–B: παροιμία τρίτον τῷ σωτῆρι, ἐπὶ τῶν τελείως τι πραττόντων. τὰς γὰρ τρίτας σπονδὰς καὶ τὸν τρίτον κρατῆρα ἐκίρνων τῷ Διὶ τῷ σωτῆρι. τέλειος γὰρ ὁ τρία ἀριθμός, ἐπειδὴ καὶ ἀρχὴν καὶ μέσον καὶ τέλος ἔχει, καὶ πρῶτος οὗτος τῶν ἀριθμῶν ἀρτιοπέριττος. τέλειος δὲ καὶ ὁ Ζεύς, ὥστε κατὰ λόγον τρίτον τῷ Διὶ σπένδεταί τε καὶ ὁ κρατὴρ τρίτος τίθεται.

24 As István Bodnár has suggested, the cult practices referred to by Aristotle may include the use of epithets like *trismakar* and *trisolbios*, which expresses the completeness and finality of the beatitude by using the number three.

all, as far as we are aware, there is no other clear early evidence for this particular Pythagorean view.[25] And it might be even in contrast with what Aristotle says about the perfection and completeness of the number *ten* in the *Metaphysics*: 'The number ten is thought [by the Pythagoreans] to be perfect (*teleion*) and to comprise the whole of the nature of numbers' (Aristotle, *Metaph.* A.5 986ᵃ7–8).[26]

Some commentators suggest that the alleged Pythagorean view alluded to in *Cael.* I.1 about the number three had its source in an Orphic hymn to Zeus. The relevant verse is quoted in the Derveni Papyrus (17.12) in the form Ζεὺς κεφαλή, Ζεὺς μέσσα, Διὸς δ'ἐκ πάντα τέτυκται, and so also in the Ps. Aristotelian *De Mundo* and Porphyry, whereas it is alluded to by Plato in *Laws* 4 715ᴇ (ἀρχὴν αὐτὸς ἔχων καὶ μέσσην ἠδὲ τελευτὴν, ὡς λόγος ἀρχαίων). Burkert has argued against the hypothesis, advanced by Tannery and Roscher, that there was an Orphic numerology that pre-dated the Pythagorean, and the Pythagoreans adopted parts of this assumed Orphic number symbolism on this point. Indeed, there is no evidence for an early Orphic numerology, and the verse about Zeus has no mention of three (or any other number). Burkert, on the other hand, maintains that 'in this respect Pythagoreanism is dependent on purely religious, or "Orphic", sources' (Burkert 1972, 467 n. 6).[27]

We are not entirely convinced about the necessity to connect the Pythagorean view as reported by Aristotle with the Orphic verse. It is remarkable that, as we have just seen, other ancient sources can perfectly well forgo the reference to the

25 These sentences from *Cael.* I.1 constitute the only piece of evidence that Burkert evokes when he comes to the number three in his discussion of Pythagorean numerology (Burkert 1972, p. 466; cf. p. 265 and p. 474). Similarly, this is the only relevant Pythagorean document that Usener discusses in his book-length study *Dreiheit*. Cf. Usener (1903).

26 But see our remarks on Moderatus in Appendix 2. Wildberg (1988), pp. 20–21, n. 57 tries to connect this doctrine with the evidence from the *Metaphysics* in the following way. In *Metaph.* A.8 990ᵃ27–29 Aristotle asks whether the number present in the *ouranos* is the same as the one present in *doxa*. Moreover, there is some evidence for taking the number of *doxa* to be 3 (cf. Aet. I. 3.8, Ascl. *in Metaph.* 34.30–31, Theo Smyrn. 98.3 in Ross 1924, 144). If so, Aristotle's question in the *Metaphyics* might be reformulated in the following terms: is the number present in *ouranos* the number 3? And this in turn could be squared with the view ascribed to the Pythagoreans in *Cael.* I.1. We find this suggestion implausible. First, the evidence for the number of *doxa* is late; moreover the same sources also register an alternative tradition according to which the number of *doxa* is not 3 but 2. Furthermore, we could not find any sign that Aristotle had knowledge of the tradition linking *doxa* to 3. Finally, it is not evident that the number of *ouranos* should also be the number of the all and all things.

27 Cf. also Moraux (1965), pp. xxx–xxxi: 'Les Pythagoriciens, transposant une vieille formule orphique relative à la divinité, considéraient comme la caractéristique d'un tout le fait de comporter début, milieu et fin et ils en déduisaient l'éminent dignité de la triade, nombre de la totalité'.

otherwise widely known Orphic verse about Zeus in connecting three and all through the trio of beginning, middle, and end, even in a context where the main *explanandum* is an epithet of Zeus. More generally, we are not convinced that one needs to pass through the Orphic verse about Zeus to realize the relationship between completeness and having a beginning, middle, and end. Or should we think that Aristotle, when he writes in the *Poetics* that '… a whole (*holon*) is that which has beginning, middle, and end' (Aristotle, *Poetics* 7 1450^{b}26–7), is also adapting the Orphic verse?

Thus, we believe that the Orphic verse does not form the immediate background of this doctrine.

Moreover, an overview of the related texts (see Appendix 2) delivers no early evidence to the effect that there were actual Pythagoreans holding the view Aristotle attributes to them. There are however some indications, coming from Aristoxenus, that even if not the precise doctrine Aristotle mentions, at least related views were held by some Pythagorean contemporaries of Aristotle. More important for our purposes, there is some – admittedly later – evidence that suggests that the doctrine as we find it in Aristotle circulated in the early Academy as a Pythagorean tenet. Indeed, this last element might help to answer the question we started with: Why does Aristotle think that it is a good idea to build his whole argument on this somewhat shaky foundation? Indeed, already Simplicius calls attention to the peculiarity of Aristotle's procedure: 'It is worth noting that Aristotle has uncharacteristically made use of Pythagorean proofs (*endeixis*) in the service of demonstration (*apodeixis*)' (Simplicius, *in Cael.* 9.10–11, transl. Hankinson, 2002). Clearly, Aristotle is usually highly critical of Pythagorean numerology, and, to the best of our knowledge, nowhere else bases an argument on such a Pythagorean doctrine.

Without solving this puzzle, we would like to suggest a few considerations that might mitigate the apparent strangeness of what Aristotle is doing here. First, as Burkert has noted, Aristotle is not using here the terminology that he otherwise uses when reporting the relation the Pythagoreans postulate between numbers and things. For elsewhere he either simply uses an '*esti*' of identification, or an '*ex*' of constitution, whereas here he says that 'the whole and all things are *delimited* (or *defined*, *hôristai*) by the three' (268^{a}11) and that 'end, middle and beginning *have* the number of the whole, which is that of the triad' (268^{a}11–13) – formulations that come with a far less heavy ontological baggage.[28]

28 Burkert (1972), p. 265, with p. 31, n. 15. This distinction seems to be ignored by e.g. Wildberg (1988), p. 20: 'For first, the Pythagoreans held that "the All (*pan*) and everything (*panta*) are determined by the number three; for end, middle, and beginning possess the number of the All, Second, there is no way the Pythagoreans could be used as reliable experts when one is discussing, say, the nature of male, female, marriage, and so forth, simply because, for Aristotle, male, female, marriage, and so forth, are not numbers and are not constituted, nor yet defined, by numbers. As opposed to this, he himself thinks that number, the number three, is in fact essential when it comes to body and dimensions. So even if the Pythagoreans are fundamentally mistaken about other things for which number is irrelevant, we might listen to them when we are discussing a phenomenon for which number is indeed crucially important. And we can do so without taking on their wacky number-based ontology. Just as importantly, in other cases the Pythagoreans leave far behind the *phainomena* in their arcane numerological speculations, whereas in this specific case their view is in full agreement with the *phainomena* and with what most people think, as it transpires from the shared religious and linguistic practices that Aristotle also alludes to.

Indeed, the conceptual connection between being a whole, having beginning, middle and end, and the number three seems to have had some general plausibility. We find the same doctrine stated without much discussion also in Plato's *Parmenides*:

> Then what about this: if it is a whole, would it not also have beginning, middle, and end? Or is it possible for something to be a whole without these three? Should anyone of them be lacking to a thing, will it still consent to be a whole?
> It will not.
> Then unity, it seems, would have beginning, middle, and end.
> Yes. (Plato, *Parm*. 145A5–B1, transl. Allen 1997)

Thus, for Aristotle to assume such a doctrine might have been natural due to its credentials. It seems to be a supposition that had currency even beyond the narrow confines of Pythagorean numerology.

Third, and closely connected to our previous point, this strategy might be also dialectically fruitful if the primary target audience is the early Academy. For, if our survey of the origin of Aristotle's attribution has some plausibility, Aristotle might have used this argument because his target audience, the philosophers in the Academy, knew this argument and presumably accepted it as authoritative.

and this is the number three", a10–13. This remark may be taken as a reference to the characteristically Pythagorean method of identifying physical and non-physical items with numbers'. Incidentally, both verbs might allude to Philolaus' theory, for he not only uses the expression 'having number' (DK B4), but also makes 'delimiting' central to his theory.

Aristotle's argument can then show with special force to that audience that the conclusions follow from premises they themselves might accept.[29]

Cael. I.1 268ª20–ᵇ5: Completeness

After having established the connection between ‘three’ and ‘all’, Aristotle introduces in 268ª20–268ª28 a further thought. Building on the results of the previous part, he argues that it is not only the case that there cannot be more than three types of extended magnitude, but also that among these types ‘body’ is exceptional, since it is complete and perfect (*teleion*). Aristotle thus introduces the idea of a hierarchy among the types of extended magnitudes on top of which we find ‘body’.

Stylistically, after the elevated tone of the previous paragraph, Aristotle switches to a rather dry and technical language which may further contribute to the view that the previous part originated from a different text.

The thought Aristotle puts forward, however, is no less intriguing. For he justifies the claim that there is a hierarchy among the types of extended magnitudes with the puzzling remark that ‘the “every” and the “all” and the “complete” do not differ in their form, but only in their matter and the things they are predicated of’ (Aristotle, *Cael.* I.1 268ª20–23).

We begin to address this difficult sentence with an observation concerning the translation. ‘Complete’ is our translation of the Greek word ‘*teleion*’. The Greek word, however, undoubtedly has a normative aspect as well. For according to the entry ‘*teleion*’ in *Metaphysics* Δ.16 the notion means both something that includes all its parts (i.e., something complete) and something that with respect of the excellence proper of its kind cannot be surpassed (i.e., something perfect).[30] Both senses, we claim, play a role in *De Caelo* I.1. Insofar as body is divisible and extended in *all* the dimensions in which a magnitude can be extended and divided, body is a complete magnitude. This accords with the first meaning of *teleion*. But insofar as no further magnitude can surpass body, body is the perfect magnitude according to the second meaning of *teleion*. We think that Aristotle deliberately exploits the two facets of the term. Wildberg thinks that to attribute an axiological meaning to *teleion* in this context is ‘philosophically absurd’

29 Note that this might be especially relevant if, as suggested, e.g., by Gigon (1952), p. 119 and others, this part of the text comes from, or is based on, an early treatise by Aristotle.

30 Cf. Aristotle, *Metaph.* Δ.16 1021ᵇ12–1022ª3. See also, Aristotle, *Cael.* II.4 286ᵇ18–23, *Ph.* II.6 207ª8–14, *Metaph.* I.4 1055ª10–16.

because it is 'simply false to say that a body *qua* body is perfect' and 'Aristotle never wanted to claim this'.[31] Wildberg does not say why he thinks that it is philosophically absurd to assume this view. Yet, we think that Aristotle wanted to claim precisely that and that there is no absurdity involved. For perfection means – we will consider Aristotle's arguments in a moment – that body cannot be surpassed in the relevant sense. Aristotle does not only say that there are as a matter of fact not more than three dimensions, but that there cannot be more than three dimensions. This latter modal claim has for Aristotle, we suggest, an axiological note. And this is why Aristotle later says that lines and surfaces are deficient.[32]

That fact that something complete and perfect cannot be surpassed is, we believe, also the key to Aristotle's rather puzzling statement that 'the "every" and the "all" and the "complete" do not differ in their form, but, if [they differ at all, they differ] only in their matter and the things they are predicated of' (Aristotle, *Cael*. I.1 268^{a}20–23). We suggest that Aristotle means that 'every' and 'all' and 'complete' are synonymous, but are predicated of different items.[33] The crucial thought is that the notion of an 'all' is logically tied to being complete and perfect.

31 Wildberg (1988), p. 22.

32 We agree therefore with Falcon's view that *teleion* conveys both the idea of completeness and that of perfection, but we disagree with his opinion that this is required by Aristotle's particular conception of the All as a 'causal system of interconnected bodies' (Falcon 2005, p. 35). Aristotle does not mention causality at all in the first chapter. Rather, Aristotle argues that both the particular bodies and the cosmos are complete and perfect magnitudes insofar as they are the only three-dimensional magnitudes and no further magnitudes can exceed them. The cosmos, as we will see, is also complete and perfect without further qualification insofar as nothing outside it exists. But the difference between the cosmos and particular bodies is spelled out in terms of the notion of 'delimitation', rather than in terms of causality. Both particular bodies and the cosmos are *teleia* magnitudes insofar as they have all possible dimensions and no further magnitudes can surpass them.

Besides its lack of textual support there is another problem in taking causal unification as a criterion of perfection as Falcon does. If one takes causal unification as a criterion of perfection it seems that particular bodies, like animals, are more perfect than the cosmos in Aristotle's system. The causal unity of an animal is higher than the unity of the cosmos, because the cosmos comprises independent substances and it seems one of Aristotle's main metaphysical tenets that a (true) substance cannot be composed of substances (cf. Aristotle, *Metaph*. Z.13 1039^{a}3–14). Thus, the cosmos has substances as parts, whereas animals do not have substantial parts. This is a difference to Plato who in the *Timaeus* argues that the cosmos is an animal. Therefore, we find it unconvincing to extract this idea from Plato and the *Physics* of Aristotle, as Falcon (2005), p. 35 does. We do not wish to deny that the cosmos is some sort of unified causal system (for a defense see Matthen, 2001). But we do deny that it is causally more unified than particular substances.

33 This reading is suggested by Wildberg (1988), p. 22, who quotes Aristotle, *Cael*. I.8 276^{b}2.

Regarding the first part of the sentence it is plausible to assume that the phrase κατὰ τὴν ἰδέαν means sameness of definition. This is reinforced by two parallel passages in which the phrase occurs.[34] Thus Aristotle says that the terms 'all', 'every' and 'complete / perfect' have the same meaning.

But what is it for these items to differ with respect to their matter and what are they predicated of?[35] Clearly, Aristotle cannot mean that those terms have themselves matter. We suggest that the second conjunct (what those terms are predicated of) is an explanation of the first (the matter of those terms). For two terms to differ in their matter is for those terms to be predicated of different items. Alexander (according to Simplicius, *in Cael.* 9.5–8) apparently had a similar interpretation. He maintains that 'every', 'all' and 'complete' are the same in form but not with reference to their objects (*hupokeimena*) because 'every' is predicated of a determinate quality, 'all' of continuity, and both of the complete.[36]

To illustrate it a little further let us stipulate that 'all' is predicated of masses and 'every' of countable items. Consider the meaning of 'all' in the sentence 'She poured all the water out' and the meaning of 'every' in the sentence 'Every person in the room drank a martini'. Arguably, the meaning of 'all' and 'every' is the same, if we allow talk of meaning in these cases. In both cases 'all' and 'every' are universal quantifiers. They refer to or pick out a certain whole. But the *hupokei-*

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34 Cf. Aristotle, *Cael.* I.8 276ᵃ32–b4: 'Moreover each of the bodies, fire, I mean, and earth and their intermediates, must have the same power as in our world. For if those elements are named homonymously and not in virtue of having the same form as ours, then the whole to which they belong can only be called a world homonymously' (Stocks' transl. in Barnes 1984, p. 452); *EN* V.1 1129ᵃ27–ᵇ1: 'Now 'justice' and 'injustice' seem to be ambiguous, but because the homonymy is close, it escapes notice and is not obvious as it is, comparatively, when the meanings are far apart, e.g., (for here the difference in outward form is great) as the homonymy in the use of *kleis* for the collar-bone of an animal and for that with which we lock a door.' (transl. Ross in Barnes 1984).

35 We want to thank István Bodnár, whose suggestions helped us a lot to improve this paragraph.

36 That is to say, whatever the term 'complete' can be predicated of, 'all' and 'every' can be predicated of, too. Thus Alexander seems to maintain that there is a difference in matter between 'all' and 'every', but 'complete' encompasses the matter of the other two terms. Besides the solution of Alexander, which we more or less follow, there is also a different exegesis by Leggatt (1995). Leggatt (1995), p. 174 proposes the following reading: all three terms signify the cosmos, but from different 'stand-points': 'every' from the stand point of the multiple bodies that constitute it, 'all' from the perspective of these bodies as a unity and 'the complete' from the view-point of the extremity of the cosmos. The main objection to this reading is that apparently 'all', 'every' and 'complete' describe the same 'material' entity (namely, the cosmos), but in different ways (i.e., with different descriptions). That seems to be the opposite of what Aristotle states in the passage.

mena of the terms are different since one is said of a mass, i.e., water, and the other of countable items, i.e. persons.

Aristotle unfortunately gives little or no indication what he thinks the *hupokeimena* are. Any suggestion is therefore bound to be speculative. A closer look at the sentence, however, shows that these speculations are unnecessary. Aristotle explicitly says that '*if* those terms differ, they differ in their matter' (268^a22). This, of course, leaves it open whether Aristotle thought they differ. Aristotle leaves – maybe deliberately – the question open. Of course, he says that any difference between these terms must be a difference in their matter. However, since the function of the sentence at hand is to justify the application of the term 'perfect / complete' to bodies, Aristotle's argument would still go through. For the crucial point in the argument is that 'all', 'every' and 'complete / perfect' have the same meaning. That is to say, the function of 'all' in an arbitrary sentence of the form '… all …', the function of 'every' in an arbitrary sentence of the form '… every …' and the function of 'complete / perfect' in an arbitrary sentence of the form '… complete / perfect …' is the same. Hence, a predication of the term 'all' is equivalent to a predication of the term 'complete / perfect'. By granting that the terms may differ with respect to their *hupokeimena* Aristotle concedes that in some cases a substitution may not be possible because there are (additional) rules of what items the terms are predicated of. It may be, to return to our example, that in many sentences one cannot substitute '… all water …' with '… every water …'. But this does not imply that 'all' and 'every' differ in their meaning.

We suggest interpreting Aristotle's remarks here in the light of a possible dispute with a Platonic theory.

A Platonist would ascribe perfection to the cosmos because of its perfect shape (or because it contains all the regular solids each of which exhibits a beautiful shape) and deny the applicability of the term 'perfect' to any old three-dimensional entity. Merely being three-dimensionally extended does not convey perfection to anything. Even the *chōra* is three-dimensionally extended, but is in no way perfect from the perspective of a Platonist.[37] But if the Platonist grants that the three dimensions are 'all' dimensions, Aristotle's reply is that the meaning of

37 It is less clear whether Aristotle would say that the *chōra* is perfect. Aristotle thinks that bodies have sharp boundaries, their surfaces. A body can thus be seen as a composite of matter and form where the matter is the extension and the form the surface that bounds the extension. Accordingly, Aristotle sometimes *defines* bodies as being bounded by surfaces (cf. Aristotle, *Ph.* III.5 204^b4–7). Since the *chōra* appears to be without a defining limit, it is not clear whether it passes the criterion of being a body. In his discussion of place in *Physics* IV.2 Aristotle seems to suggest that the Platonic *chōra* corresponds to the *matter* or *extension* of a body. Cf. Aristotle, *Ph.* IV.2 209^b1–16). We would like to thank Chris Noble for pointing this out to us.

'all' and 'complete / perfect' is the same and hence it is justified to apply the term 'complete / perfect'. And this, as we have seen, may also be derived from Pythagorean premises.

Once the connection among 'three', 'all', and 'complete / perfect' is established the main argument of our chapter is completed: Only 'body' is complete and perfect among the magnitudes, since it is defined / determined by the three, which in turn means that body is defined by 'all'.

The structure of the argument can thus be represented as follows:

(1) Magnitudes are defined by the number of the dimensions in which they are extended and divisible.
(2) Body is defined by 'three'.
(3) 'Three' implies 'all'.
(4) 'All' implies 'complete and perfect'.
(5) Hence, body is complete and perfect.
(6) The other magnitudes, in contrast to body, are not complete and perfect, since they are defined by 'one' (line) or 'two' (surface) respectively.

That Aristotle's ultimate conclusion of the chapter is the perfection and completeness of body is endorsed by the next lines in which he makes some additional comments on his results so far and completes his argument ($268^{a}30$–$^{b}5$) by showing that there cannot be a further magnitude over and above body.[38] Since

38 Lines $268^{a}24$–30 introduce hardly any new ideas. First, Aristotle argues that being divisible in three ways is divisible in all ways ($268^{a}24$–28). The number of dimensions and the number of ways in which a magnitude can be divided correspond. Then he raises a problem concerning the equivalence of being divisible and being extended. It is not difficult to understand what Aristotle says, but rather why he says it. What he says can easily be stated: All magnitudes that are divisible are continuous (divisibility implies continuity), but whether the reverse is true is not yet clear. Commentators usually refer to *Physics* VI.1, where Aristotle proves the reverse. But what up to now none of the commentators has quite noted is the impact this assertion has on the very beginning of Aristotle's discussion of body in lines $^{a}6$–8. These lines are usually read as a definition: The continuous $=_{\text{def.}}$ that which is divisible into ever-divisible parts. But on the grounds of $^{a}28$–30 this cannot be a definition, because it would be nonsensical to define the continuous as the ever-divisible and then go on to ask whether the continuous is in fact divisible. The connection must be looser. So the first step is to see that continuity is not defined by divisibility. All Aristotle is maintaining in the first passage is that divisibility implies continuity, that is, it is sufficient but not necessary.

One could speculate why Aristotle does not want to establish a stronger connection. One reason could be that he might not want to assert too much (for example, is time really ever-divisible?). Another, possibly simpler, reason is that logical rigour forbids him to do so. If the definition of the continuous does not make overt reference to divisibility (which it indeed does not if we follow *Cat*. 6 and *Ph*. V.3), one must prove that continuity implies divisibility, rather than body is complete and, in virtue of being complete, it does not allow a further augmentation, a 4-D entity is impossible:

> One thing, however, is clear. There is no transition to another kind of magnitude, as we passed from length to surface, and from surface to body. For if we could, it would cease to be true that body is complete magnitude. We could pass beyond it only in virtue of a defect in it; and that which is complete cannot be defective, since it is in all ways. (*Cael.* 1.1 268a30–b5, transl. Stocks in Barnes 1984, modified)

This is a highly compressed argument and we underlined what we take to be its crucial step. To facilitate a better understanding let us present it in a stepwise reconstruction. The argument has three premises from which the conclusion that there is no transition from body to another genus follows:

(1) A transition to another genus is possible only if there is a deficiency. (Premise)
(2) Something is defective if and only if it is not complete. (Premise)[39]
(3) Body is complete. (Premise)
(4) Body is not defective. (From 2 and 3)
(5) Hence, there cannot be a transition from body to another genus. (From 1 and 4)

The underlined sentence thus describes a counterfactual situation: If there were a transition from body to another genus, body would not be the complete magnitude. But since we know that it is true that body is complete (because it instantiates the three, and three is complete), we can conclude by *modus tollens* that there is no transition.

We wish to flag that Aristotle again relies strongly on the premise that there cannot be more than three dimensions. This once again emphasizes the extent to which Aristotle relies on the argument taken from the Pythagorean view together with cultic and linguistic practices.

The intellectual background is manifest from the doctrine Aristotle alludes to here, too. For he not only denies that there is a transition from body to another genus, but he apparently assumes that there *is* a transition from line to surface and surface to body.

just asserting it. However, if divisibility is only a sufficient condition it is unclear why Aristotle brings up the topic of divisibility at all. What seems important is extension in three dimensions, rather than divisibility. Commentators usually stress the anti-Platonic or anti-Atomist stance, but a satisfactory solution has not yet been proposed.

39 The argument requires, strictly speaking, only the weaker premise that if something is defective it is not complete. However, we believe that Aristotle endorses the stronger version.

What type of transition has Aristotle in mind here? The word *ekbasis* occurs only here in the *Corpus Aristotelicum* and *metabasis* usually describes the elemental transformations[40] (cf. Aristotle, *Cael.* III.1 298ᵇ1, III.7 306ᵃ32, a reference to the Atomists). Hence, in their more usual senses these words describe a physical transition, which makes it unlikely that Aristotle wants to restrict the processes referred to here to a mathematical transition.[41] We suggest that the passage should be understood in the context of a no less intriguing passage in Plato's *Laws*: 'What happens when the generation of all things occur? Clearly, an *arkhê* takes up growth, and reaches a second stage (*metabasin*) and then the next one out of this second, so that as soon as it reaches the third, there is something for percipient things to perceive' (*Laws* X, 894A).

We think that this parallel is crucial. First, *metabasis* here clearly refers to the passage from n dimension to n+1 dimension. We are moreover in a Pythagoreanizing context. Third, the last clause unambiguously guarantees that the text describes the generation of physical, perceptible bodies. Hence, it provides important external evidence against Leggatt's (Leggatt 1995, p. 174) suggestion that in *De Caelo* I.1 Aristotle speaks about the generation or mathematical construction of geometrical objects as opposed to physical bodies. Finally, the parallel with *Laws* X also shows that Aristotle is referring here to a doctrine which he may or may not subscribe to, but which certainly does not originate with him, and which had currency among the assumed target audience of the work. So we suggest to understand the purport of the passage in the following way: Even if someone thinks, as the Pythagoreans apparently do, that there is a transition from n-1 dimension to n dimension leading up to physical bodies, this person is also required to agree that the process must stop at the third dimension given that three comes with completeness, as all people agree and the Pythagoreans themselves explicitly teach. If so, they also need to agree that bodies are complete / perfect.

Moreover, the idea of a quasi-natural process of generation on top of which we find bodies also hints at a connection between the substance and nature of physical bodies and their being three-dimensional. In virtue of being three-dimensional bodies are perfect and complete, but being three-dimensional is not the substance of physical bodies. It is a quantitative feature of them.[42] On the

40 See Bonitz, *Index*, 226ᵇ10, 459ᵃ15–31.

41 This is especially true, if one considers the fact that up to now Aristotle has nowhere drawn such a distinction and the context at hand is physical science. Contra Wildberg (1988), p. 25.

42 We admit that Aristotle never says that in so many words. But we believe that it is indeed a consequence of the famous stripping argument in *Metaphysics* Z.3: 'For while other things are other hand, the completeness of bodies is not established without taking into account the nature of bodies. Logically surfaces are prior to bodies, because surfaces can be defined without reference to bodies, but not *vice versa*.[43] Hence, the argument for the priority of bodies is grounded in considerations about the nature of bodies. If the process leading towards bodies is seen as a quasi-natural generation, then this process leads to something that is prior by nature. It is the nature of bodies, we suggest, that explains their three-dimensionality. *Because* bodies have the nature that they in fact have, they must be three-dimensional. Their being three-dimensional is due to and caused by their nature.[44]

Be that as it may, Aristotle's argument that bodies are perfect is in the same vein also an argument that there are only three dimensions in physical space. Aristotle presents here the first argument in the history of Western philosophy for this conclusion. Even though it is based on some shaky evidence, it is remarkable that Aristotle gives a justification at all. The first attempt to prove in a more formal manner the necessity of three-dimensionality seems to reach back to Ptolemy as Simplicius tells us in his commentary *ad locum*:

> The estimable Ptolemy beautifully demonstrated in his single volume *On Dimensions* that there are no more than three dimensions from the fact that dimensions must be bounded, and dimensions are bounded in respect of the taking of straight perpendiculars, while it is only possible to take three straight lines at right-angles to each other, two according to which the plane is defined, the third measuring depth. Consequently, if there were another dimension after the third it would be utterly unmeasured and indeterminate. Thus Aristotle seems to have established that there is no transference to another dimension by enumeration of instances, while Ptolemy demonstrated it. (Simpl. *in Cael.* 9.21–29, transl. Hankinson 2002)

We shall not further comment on Ptolemy's proof since it lies outside the scope of our paper. Instead let us make a brief comment about the style in which the two proofs proceed. Ptolemy's argument is interestingly different from Aristotle's since it takes into account the specific nature of magnitude rather than being an argument about the perfection of the number three in general. Aristotle presents an argument to the effect that the number three is perfect and concludes that

attributes, products, and capacities of bodies, length, breadth, and depth are quantities and not substances (for a quantity is not a substance)' (*Metaph.* Z.3 1029ᵃ12–15). If one considers physical bodies solely without their characteristic capacities, one considers a quantity and not a substance. Moreover, if being three-dimensional were the substance of something, mathematical objects would be substantial, too.

43 Aristotle, *Metaph.* B.5 1002ᵃ2–10; *Metaph.* Δ.8 1017ᵃ17–21.

44 In this context cf. *Metaph.* M.2 1077ᵃ24–31.

in the case of dimensions the number three indicates perfection, too. Ptolemy, by contrast, begins with the way in which distances should be defined and concludes that from that definition it follows that there cannot be more than three dimensions. The difference between Aristotle and Ptolemy is precisely not the difference between a deductive and an inductive argument, as Simplicius wants to have it. The difference rather lies in what we may call a topic-neutral or topic-specific approach. To us it may seem natural to take the topic-specific approach, as Ptolemy did, simply because it seems plausible that from general considerations about the number three nothing of relevance follows in the case of the dimensions.[45] But from that it does not follow that Ptolemy's argument is much better. For Ptolemy's argument is based on a *petitio principii*. To say that there can only be three perpendicular lines is to presuppose three-dimensional space. Obviously, in a four-dimensional space there would be four such lines. Thus, what can be considered as a weakness in Aristotle's argument is his reliance on frail arguments concerning the number three. But his general method of a topic-neutral approach is not faulty in the same way. Hence, we think that one must give Aristotle credit for being the first thinker in the history of Western thought to see the need for a justification or explanation of the three-dimensionality of physical bodies and space at all. Even though his arguments might not stand up to the highest expectations, or even to more modest expectations, it is utterly remarkable that Aristotle should have felt the need for an argument. We shall come back to this point in our conclusion.

45 A formidable and polemical argument to that effect is presented by Galileo at the beginning of his *Dialogue Concerning the Two Chief World Systems*, where his spokesman Salvatius says:

'To tell you the truth, I do not feel impelled by all these reasons to grant any more than this: that whatever has a beginning, middle, and end may and ought to be called perfect. I feel no compulsion to grant that the number three is a perfect number, nor that it has a faculty of conferring perfection upon its possessors. I do not even understand, let alone believe, that with respect to legs, for example, the number three is more perfect than four or two; neither do I conceive the number four to be any imperfection in the elements, nor that they would be more perfect if they were three. Therefore it would have been better for him to leave these subtleties to the rhetoricians, and to prove his point by rigorous demonstrations such as are suitable to make in the demonstrative sciences' (transl. Drake 1967).

Bodies in the form of parts (elements) and the cosmos (*Cael.* I.1 268^{b}5–10)

The remarks about the sources of perfection lead us finally to the main topic of *De Caelo*, the cosmos. In the last lines of the chapter Aristotle argues that the cosmos is complete in yet another sense: it is complete in virtue of being a totality. The cosmos is all-encompassing and is not bounded by something from the outside. This sets it apart from the particular bodies, which have a delimitating effect on each other.

Aristotle ascertains that bodyhood implies completeness or perfection in one sense, but not necessarily in every sense. For bodies that belong to the class of parts, Aristotle says, are complete in the sense that they are determined, defined, or delimited (*hôristhai*) by the number three by being extended in exactly three dimensions. Yet at the same time they are also determined, defined, or delimited by another, undetermined multitude or number, distinct from the number three, insofar as they are touched from the outside by an undetermined number of things. In this sense they are defined or delimited by a numerosity distinct, or at least not necessarily equal to three, and are in this sense not perfect or complete. They are 'many' (*polla*), constitute an undetermined multitude, as opposed to being perfect, complete, and all or whole (*pan*). This reasoning however does not apply to the cosmos because it is not delimited or determined by anything touching it from the outside, and therefore it is only determined by the number three in the sense we considered in the previous part of the chapter. The cosmos (*to pan*), as its name also shows, is complete, perfect, and a *pan* in an unrestricted sense.

Understanding the reference of the term 'bodies that are in the form/class/species (*eidos*) of part' (Τῶν μὲν οὖν ἐν μορίου εἴδει σωμάτων, 268^{b}5) is crucial for any interpretation of the passage. Most commentators assume without further ado that the reference is to physical particulars, i.e., ordinary individual physical objects, trees, dogs, frisbees, and the like. There are however multiple problems with this reading. First, it is quite un-Aristotelian to say that such physical substances are determined, defined, or even delimited by 'touch'. This would imply that they lack intrinsic unity,[46] and their individuation is dependent on their neighbours. Second, it is even more problematic, on Aristotelian, or indeed on any other, ground to say that these physical bodies become many in some sense by touch. True, if the frisbee touches the table by its bottom side, and touches my hand with its upper side, one might say that it has become many in the sense that

46 Cf. Matthen (2001), p. 173.

we can distinguish between its table-touching lower and hand-touching upper side. But this is a very weak sense of becoming many.

We suggest a significantly different understanding of the passage based on a different understanding of the reference of the term 'bodies that are in the form/class/species of part'. A confirmation comes from the immediate sequel of the text. For already in the first paragraph of chapter two, immediately following our sentence, the term 'bodies that are its parts with respect to their form' (περὶ δὲ τῶν κατ' εἶδος αὐτοῦ μορίων) (268^{b}13) refers to the five elements. This is also how Simplicius *ad loc*. understands the reference of the phrase.[47]

Once this is recognized, the road is open to a different, and as we see it more plausible, understanding of the problematic claim that these bodies are determined and delimited by touch, and thereby become many. We suggest the following. The elements are denoted by mass terms, and in this sense particular portions of them are individuated not by some intrinsic feature of the portion of stuff in question. A certain portion of water is individuated not by some intrinsic feature of it, but rather by its being surrounded by portions of earth, air, etc.

In this way, we understand the phrase διὸ τρόπον τινὰ πολλὰ τῶν σωμάτων ἕκαστόν ἐστιν (268^{b}7–8) not in the sense that each individual physical object, except the cosmos, is many, but rather that each of the four elements is many in so far as there are individual portions of each of them, each such portion being delimited or defined from the outside by portions of other elements.

Our reading is further confirmed by the following parallel passage from the third book of *De Caelo*:

> It is manifest that the simple bodies are often given a shape by the place in which they are included, particularly water and air. In such a case the shape of the element cannot persist; for, if it did, the contained mass would not be in continuous contact with the containing body; while, if its shape is changed, it will cease to be water, since the distinctive quality is shape. Clearly, then, their shapes are not fixed. (*Cael.* III.8 306b9–15, transl. Stocks in Barnes 1984)

Aristotle argues against theories which assign specific shapes to the simple bodies. One of his arguments against such a theory is based on the observation that the four elements are shaped by their surroundings. The single portions of water are shaped from the outside. They lack intrinsic unity and are dispersed. The cosmos in contrast to that is complete in every respect. It is a totality without further limitation or delineation from without.

47 Simplicius, *in Cael* 11.27–30. Cf. Matthen (2001), p. 178.

Conclusion

The chapter is densely argued and it contains some *bona fide* Aristotelian tenets and arguments familiar from the *Physics* and the *Metaphysics*. Yet, these arguments are mixed in a seemingly curious way with arguments from authority, as well as with references to religious, cultic, and linguistic phenomena. The outcome is a baffling brew, quite unique in its air and flavour in the whole Aristotelian corpus.

The emphasis, all through the chapter, is on the perfection and completeness of bodies. Perfection has, of course, a strong axiological undertone and is an attribute that belongs to divine beings; indeed, the text at some points starts to read like a prose hymn to bodies. This quasi-theological aspect of the text is reinforced by the arguments which seek to establish, via the perfection of the number three, the perfection of bodies. It is remarkable that Aristotle can formulate his ultimate conclusion about the supreme perfection of the cosmos, so to speak, from the bottom up. The cosmos is perfect not because some divinity created it to be perfect, and not because it is itself a sentient divine being, as Timaeus teaches, but because it is a body, that manifests perfection to an even higher degree than other bodies do. It is just as remarkable in this respect that even if the cosmos does manifest perfection at a higher degree, this perfection, as we have seen, is not of a fundamentally different type of perfection than the one manifested by other bodies constituting the cosmos as a whole.

Conversely, parts of the cosmos are not perfect in so far as, or because, they inherit something of the perfection of the whole which they are parts of – no, they manifest perfection in and of themselves, by the very fact that they are bodies.

Incidentally, all this emphasis on the perfection of bodies implicitly, but necessarily, raises the value of physics or natural science as such, in so far as its proper objects are such perfect beings. And of course, it even further raises the value and importance of the present treatise in so far as its subject matter is such a supremely perfect being.

All these conclusions about the perfection of the bodies, and the supreme perfection of the cosmos *qua* body, as well as the implications about the status of physics, have a strong, even if unstated, polemical edge in the intellectual context of the early Academy. Physical bodies, for Plato, are necessarily and irremediably imperfect. And the little share they have of positive attributes – beauty, regularity, orderliness – they possess because of the careful planning and creative handiwork of the demiurge. Bodies can be characterized with value attributes, and can contribute to the composition of valuable physical objects, due to their respective *geometrical forms* – a gift from the Demiurge. It is, once again, crucial to realize that bodies, according to the arguments put forward in *De Caelo* I.1, are perfect irrespective of their other properties, geometrical or otherwise.

Aristotle is thus putting forward in our chapter a novel assessment of bodyhood, based on the criterion of three-dimensionality, together with a novel conception of perfection and completeness, based on the perfection of the number three, and applies it to the cosmos as a whole.

Appendix 1

The sentence at 268ᵃ4–6 offers two alternative ways of translation. First, one can translate: 'For among the things constituted by nature some are bodies and magnitudes, some are things that body and magnitude have and some are principles of their having <these things>' (τῶν γὰρ φύσει συνεστώτων τὰ μέν ἐστι σώματα καὶ μεγέθη, τὰ δ' ἔχει σῶμα καὶ μέγεθος, τὰ δ' ἀρχαὶ τῶν ἐχόντων εἰσίν, 268ᵃ4–6). This is how we are inclined to read the sentence. It has already been proposed by Sedley (in conversation to Ch. Wildberg (cf. Wildberg (1988), p. 18) and accepted by Sharples (1998), p. 42). Some commentators refer to this reading without endorsing it (cf. Wildberg (1988), p. 18; Falcon (2001), p. 52 n. 51 and Falcon (2005), p. 44 n. 24). Both Wildberg and Falcon follow the standard translation of the passage: 'For among things constituted by nature some are bodies and magnitudes, some have body and magnitude and some are principles of things with body and magnitude'.

The standard translation – although syntactically more natural – faces the following difficulties. First, there seems to be a discrepancy between the objects of physical science as they are listed in the first sentence of *Cael.* and the things constituted by nature as they are listed in the '*gar*' clause. For – according to this translation – there is no reference to attributes and movements of bodies among the things that are by nature (see *Cael.* III.1, 298ᵃ27–ᵇ1). The first sentence says that physical science studies bodies and their attributes. The second sentence says that things by nature are bodies and things that have body. Given that the second sentence is meant as a justification of the first, this is unsatisfying.

Moreover, according to the standard translation only things having body and magnitude would have principles. But the first items, i.e., body and magnitudes, would not. This again is problematic because we should expect that all things that exist by nature possess a principle, namely their nature. The alternative translation we suggest avoids these difficulties. Of course, the interpretation could go either way and we do not want to rule out that the standard translation can be maintained. For the choice of the translation depends on how one interprets the extension of the terms. Wildberg's reading has no problems with the notion of principle in play here, for he takes respectively (a) 'body and magnitude' as a reference to geometrical bodies, (b) 'things having bodies and magnitudes' as referring to physical bodies, and (c) 'principles of things having body and magnitude' as a reference to principles of physical bodies (cf. Wildberg 1988, pp. 17–19). This reading allows him explaining the reference to principles of things having bodies and magnitudes as a reference to principles of physical bodies only. It is however difficult to harmonize this reading with Aristotle's statement that all things mentioned here are things constituted by nature and studied by the physicist. For it is unlikely that for Aristotle geometrical bodies are constituted by nature. It seems un-Aristotelian to say that there are geometrical bodies and physical bodies side by side and both are constituted by nature. As we shall make clear shortly (see Appendix 3 below), physical bodies are by nature and the geometer studies an aspect of these physical bodies. But there are no geometrical solids over and above physical bodies. Even less geometrical bodies that are constituted by nature.

Falcon (2005), pp. 42–45, on the other hand, assumes that (a) 'bodies and magnitudes' refers to simple bodies and to the heaven and its parts, (b) 'things having body and magnitude' to living beings, and (c) 'principles of things having bodies' to the soul. According to Falcon there is no real gap between the subject matter of physical science and the things constituted by nature as they are listed in the text, since the elements of this list would explain the fact that science of nature is 'for the most part' but not exclusively concerned with bodies and magnitudes. Therefore, in the '*gar*' clause Aristotle would be quoting other things existing by nature which were not previously mentioned as the subject matter of physical science. Though this reading cannot be ruled out we think that on Falcon's reading there is too much emphasis on the phrase 'for the most part'. It seems more natural to assume that Aristotle wants to justify his claim about the subject matter of physical science (i.e. bodies and magnitudes, their affections, and their principles) with a list of things that exist by nature, rather than justifying what he has left out. Indeed, Falcon himself says the passage gives 'a compressed but adequate description of the subject matter of the science of nature' (Falcon (2005), p. 44). If that is true, then living beings, their affections, motions and principles should be among the things mentioned in the first sentence.

Appendix 2

There is surprisingly little early evidence to support Aristotle's testimony according to which Pythagoreans connect three and all. Commentators sometimes evoke in this context the incipit of the *Triagmos* of Ion of Chios: 'This is the beginning of my account. All things are three, and there is nothing more or less than these three. The excellence (*areté*) of each one thing is a triad: comprehension, strength, and fate' (DK B1, transl. based on Baltussen 2007).[48] First, we don't think that apart from the importance attached to the number three or the triad in (the constitution of) things, the content of Ion's doctrine is close to the one Aristotle attributes to the Pythagoreans. Moreover, although the doctrine has a numerological overtone, as far as one can see from this brief fragment, Ion is unlikely to have played the Pythagorean in the *Triagmos*. One indication of this is Ion's apparently critical remark about Pythagoras in the same work, according to which Pythagoras 'wrote some poems and attributed them to Orpheus' (DK B2).[49]

Although less often mentioned by commentators, more promising is a fragment by Aristoxenus.[50] According to Stobaeus (*Ecl.* I Prooem. 6) Aristoxenus in his book *On Arithmetic* wrote the following:

> A unit is a beginning of number, and number is a multitude consisting of units. Of numbers, the even are those that are divisible into equal parts, and the odd are those that are divisible into unequal parts and have a middle. They think thus that the crisis and changes of illnesses occur on odd days given that the odd has the beginning, the end, and the middle, which correspond to the beginning, culmination, and abate [i.e., of the disease].

It might be suggested that what Aristoxenus specifically has in mind is the first odd number, i.e., 3, and thereby takes 3 as the number which 'has' beginning, middle, and end. Yet the connection with our passage in *De Caelo* I.1 still seems rather strained. First, it is not entirely clear to whom Aristoxenus attributes this doctrine. (Before the part quoted, the last group mentioned specifically was that of the Egyptians.) Moreover, there is no actual mention of the number 3. Third, the context is medical, and more generally seems to refer to processes that have a beginning, a middle, and an end. Just as important, there is no mention of the all and the every, which are after all the pivotal points in Aristotle's argument. On the whole, we could nonetheless come up with an elaborate story along the following lines: Aristoxenus learned the doctrine that linked 3 and 'all' from his Pythagorean teachers, and then informed Aristotle about it, even if in this fragment he gives only some aspects, or applications of this doctrine. Even if this story must remain exceedingly speculative, the fragment of Aristoxenus suggests,

48 The fragment and its context has received a detailed examination in Baltussen (2007).
49 For further indications that Ion was critical of Pythagoras, see Dover (1986), p. 31. For a full discussion of Ion's relation to Pythagoras and Pythagoreanism, see Baltussen (2007), pp. 301–11.
50 We are grateful to Carl Huffman for calling our attention to and discussing this text with us. Cornford (1923), p. 2 n. 5 connects Aristoxenus' text to the passage in *Cael.*, glossing it by remarking that 'This sounds primitive'.

with all the caveats mentioned above, that there were contemporary Pythagorean doctrines that were *close* or *related* to what we find in Aristotle.

Significantly more distant in time, but much closer in content, is the evidence from the Neo-Pythagorean Moderatus of Gades, probably a younger contemporary of Plutarch. Porphyry in his *Life of Pythagoras* (48–53) summarizes the Pythagorean doctrines that Moderatus had expounded in his work on numbers in eleven books. Having explained that the Pythagoreans concentrate on numbers 'for explanatory purposes' (διδασκαλίας χάριν), because it is so hard to speak about the first principles directly, he turns to a patently Platonized account of the first two principles, the Monad and the Dyad. Then he continues with the characterization of the Triad (51):

> The same reasoning applies to other numbers as well, for they were all ordered according to certain powers. For there is something in the nature of things which have beginning, middle, and end; they [i.e. the Pythagoreans] denoted the form and nature which is such by the number three. Hence they said also that anything that makes use of a middle is triform [, which was stated about every perfect / complete thing]. They said that if anything was perfect / complete it would make use of this principle, and set in an orderly arrangement (κεκοσμῆσθαι) according to it. And since they were unable to use any other name for it, they applied the name of the triad to it; and whenever they tried to bring us to the conception of this principle they brought us by this form. And the same reasoning applies to other numbers as well.

This account attributed by Moderatus to the Pythagoreans is obviously very close to what we find in Aristotle. Indeed, the next paragraph, focusing on number ten, puts the correspondence into relief even more. For we learn there that the perfect number is ten (cf. Aristotle, *Metaph.* A.5 986^{a}7–8). This datum makes us more alert to the fact that the triad or number three is not said to *be* perfect / complete, but that perfect / complete things make use of, and are characterized by the three, standing for the trio of beginning, middle, and end. This corresponds very closely to the view attributed to the Pythagoreans in *De Caelo* I.1.

This obviously raises the question of the relationship between Aristotle's and Moderatus' testimony. It cannot of course be excluded that Moderatus, or his source, is dependent on Aristotle. However, in view of the strongly Platonizing tone of Moderatus' account of the first two principles of the Pythagoreans, we find the following, equally speculative, scenario more attractive. The account about the number three, roughly as we find it in Aristotle and Moderatus, was circulating in Platonist circles from the early Academy as a Pythagorean doctrine.

This suggestion might be reinforced by further Pythagoreanizing Platonist sources, and most of all the *Theology of Arithmetic,* attributed to Iamblichus. In the chapter on Three, the author takes completeness or perfection (*teleios*) to be the central characteristic of the triad, emphasizing that even though other numbers in the tetrad also show perfection in their own way, the triad is more particularly perfect than the others (τέλειός γε μὴν ἰδιαίτερον τῶν ἄλλων ἐστίν, 14.20–21). There is also reference to cult practices (16.13–15). Even closer to what we find in Aristotle and Moderatus is the report from Anatolius: 'The triad as the first odd number is called perfect by some, in so far as it is the first to indicate totality, beginning, middle, and end' (17.4–5). The report then continues by reference to cult practices, prayers and libations, without further specifying the rites in question. Of course, once again, part or whole of the relevant views might actually be based on Aristotle's text, so there is no guarantee that we are dealing with independent evidence. We are however more inclined to think that all these reports, including Aristotle's, go back to an Academic source, written or oral, that attributes this view to the Pythagoreans.

Appendix 3

Aristotle's philosophy of mathematics is most fully expounded in *Metaphysics* M.3 and *Physics* II.2. For reasons of space we cannot deal here with the intricate question how Aristotle's philosophy of mathematics is to be understood. We refer the reader to the following two papers by Mueller (1970) and Lear (1982) that frame much of the ensuing discussion. But we suggest that Aristotle had the basis of his theory already developed by the time he wrote *De Caelo*. For at the beginning of the third book Aristotle distinguishes between physical and mathematical impossibilities using the assumption that bodies are composed of planes. He says:

> But with respect to physical bodies there are impossibilities involved in the view which asserts indivisible lines, which we may briefly consider at this point. For the impossible consequences which result from this view in the case of mathematical boides will reproduce themselves when it is applied to physical bodies, but there will be difficulties in physics which are not present in mathematics; for mathematical bodies are said on the basis of abstraction, whereas physical ones on the basis of addition. (*Cael.* III.1 299a11–17, transl. Stocks in Barnes 1984, modified)

Aristotle explains his belief that mathematical theorems hold of physical entities by pointing to the fact that mathematical objects have fewer properties than physical ones. It is clear from the context as well as from Aristotle's general views on abstraction that the point of the passage is that physical and mathematical bodies can have exactly the same property, e.g. being extended. (For Aristotle's theory of abstraction see Cleary (1985), Mendell (1986), Detel (1993), pp. 189–233.) Mathematical science and physical science differ in the number of assumptions they make. For example, we may – somewhat roughly – assume that in studying bodies the mathematician presupposes only that they are extended in three dimensions. The physicist studies them insofar as they are three-dimensionally extended and have a nature, i.e., a principle of motion and rest. What is abstracted in the case of mathematical science and added in the case of physical science is the property of having a principle of motion and rest. The property of being extended is, however, present in both cases. That is, both the mathematician as well as the physicist assume that their objects are extended and divisible in exactly the same way. Otherwise one could not deduce from the mathematical impossibility that bodies are composed of planes the physical impossibility of their being so composed.

Thus we attribute to Aristotle a mathematical realism in the following sense. The basic properties of being extended and being infinitely divisible are properties that physical bodies have in a precise and realistic way. Hence, if Aristotle mentions these properties in the first chapter he cannot and does not distinguish mathematical and physical bodies by way of those properties. (Notice that we remain silent on the question whether there are perfect circles or squares in the physical world. We only state that there are perfect instantiations of three-dimensionality and infinite divisibility).

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