ERRATUM TO: ON THE COHOMOLOGY OF TORELLI GROUPS

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Abstract. We resolve two mistakes in Section 8.1 of the named paper.

A typo. On pp. 75-76 of [KRW20] (p. 52 of the arXiv version) we mistranscribed the computer-calculated Poincaré series for $H^*(B\mathrm{Tor}^+(W_g,*);\mathbb{Q})^{\mathrm{alg}}$ and $H^*(B\mathrm{Tor}^+(W_g);\mathbb{Q})^{\mathrm{alg}}$. In both cases the term $2s_{\langle 2^3,1^3\rangle}$ should instead be $s_{\langle 2^3,1^3\rangle}$. This now makes Remark 8.2 irrelevant: there is nothing to explain, as our expression now agrees with Sakasai's computation in [Sak05] (with the V_1 term present).

Relation to Sakasai's result. On pp. 76-77 of [KRW20] (pp. 52-53 of the arXiv version) we described how to settle the ambiguity in Sakasai's paper [Sak05], but the argument given is fallacious. The image of the composition

$$\Lambda^3(V_{1^3}) \xrightarrow{\tau^*} H^3(B\mathrm{Tor}^+(W_g);\mathbb{Q}) \longrightarrow H^3(B\mathrm{Tor}(W_g,D^2);\mathbb{Q})$$

after applying $[-\otimes V_1]^{\mathrm{Sp}_{2g}(\mathbb{Z})}$ is *not* the subspace of those elements which can be represented by trivalent graphs with one leg, three internal vertices, and no loops as claimed, but is instead something more complicated: see [RW21, Section 2.5]. Rather than pursue that we give an alternative, simpler, argument.

Lemma. *For $g \geq 2$ the class*

$$\kappa_{\varepsilon\cdot e^2} - \frac{1}{\chi}\kappa_{\varepsilon\cdot e}\kappa_{e^2} \in H^3(B\mathrm{Diff}^+(W_g,*);\mathcal{H})$$

is pulled back along $p\colon B\mathrm{Diff}^+(W_g,*) \to B\mathrm{Diff}^+(W_g)$.

Proof. The universal W_g-bundle $\pi\colon E \to B\mathrm{Diff}^+(W_g,*)$ with section is pulled back along p from the universal W_g-bundle $\bar{\pi}\colon \bar{E} \to B\mathrm{Diff}^+(W_g)$. In [RW21, Section 1] we have explained that there is a class $\bar{\varepsilon} \in H^1(\bar{E};\mathcal{H})$ which pulls back to $\varepsilon - \frac{1}{\chi}\kappa_{\varepsilon\cdot e} \in H^1(E;\mathcal{H})$. Thus the class $\pi_!((\varepsilon - \frac{1}{\chi}\kappa_{\varepsilon\cdot e})\cdot e^2) = \kappa_{\varepsilon\cdot e^2} - \frac{1}{\chi}\kappa_{\varepsilon\cdot e}\kappa_{e^2}$ is $p^*(\bar{\pi}_!(\bar{\varepsilon}\cdot e^2))$ as required. □

Corollary. *The map*

$$(*) \qquad \kappa_{e^2}(-)\colon V_1 \longrightarrow H^3(B\mathrm{Tor}(W_g,D^2);\mathbb{Q})$$

factors over $H^3(B\mathrm{Tor}^+(W_g);\mathbb{Q})$.

Proof. On restricting the class $\kappa_{\varepsilon\cdot e^2} - \frac{1}{\chi}\kappa_{\varepsilon\cdot e}\kappa_{e^2}$ to the Torelli group, the second term vanishes as $\kappa_{e^2} = 3\kappa_{\mathcal{L}_1} = 0$. Thus it corresponds to the map $(*)$, and it factors over $H^3(B\mathrm{Tor}^+(W_g);\mathbb{Q})$ by the Lemma. □

From this we proceed just as in the published paper: the map $(*)$ has been shown to be nontrivial, so $H^3(B\mathrm{Tor}^+(W_g);\mathbb{Q})$ contains a copy of V_1 as claimed.

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References

[KRW20] A. Kupers and O. Randal-Williams, *On the cohomology of Torelli groups*, Forum of Mathematics, Pi **8** (2020), e7. 1

[RW21] O. Randal-Williams, *Notes on twisted Miller–Morita–Mumford classes*, https://www.dpmms.cam.ac.uk/~or257/TwistedMMMNote.pdf, 2021. 1

[Sak05] T. Sakasai, *The Johnson homomorphism and the third rational cohomology group of the Torelli group*, Topology Appl. **148** (2005), no. 1-3, 83–111. 1

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