LEARNING SEMINAR ON HERMITIAN THEORY FOR STABLE ∞-CATEGORIES

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1. Organisation

· Modelled on Harvard's Thursday seminar.
· Ideally on Thursdays (for the sake of tradition).
· Two hour meetings with a break in the middle.
· One or two speakers per meeting.

2. Tentative schedule

(1) *Overview.* This will be an overview of the classical definitions, the modern definitions, and the results we will be trying to understand. There will also be a discussion of applications.
(2) *Poincaré categories* [CDH$^+$20a, §1.1–1.3]. Quadratic and bilinear functors in the context of stable ∞-categories. Hermitian and Poincaré categories, their classification. Include many examples.
(3) *Poincaré objects* [CDH$^+$20a, §2.1–2.5]. Spaces of Hermitian and Poincaré objects, the hyperbolic category, the L-groups and Grothendieck–Witt groups of a Poincaré category.
(4) *Examples: Poincaré structures on modules.* [CDH$^+$20a, §3.1–3.3, 4.2] Explain involutions and genuine involutions on modules, how the latter give rise to Poincaré categories. This may require a discussion of Tate diagonals [NS18]. Give examples from ordinary rings.
(5) *Examples: Poincaré structures on parametrised spectra* [CDH$^+$20a, §4.3–4.4] Explain visible Poincaré structures on group rings and parametrised spectra. Comment on the role of visible L-groups in geometric topology [WW14].
(6) *Cobordism categories* [CDH$^+$20b, §2.1–2.3]. Use the Hermitian Q-construction to define the cobordism category of a Poincaré category.
(7) *Additivity theorem* [CDH$^+$20b, §1, 2.5–2.6]. Explain the statement of the additivity theorem for cobordism categories and outline its proof. This will require aspects of §1.
(8) *Grothendieck–Witt spectra and the Bott–Genauer sequence* [CDH$^+$20b, §3.3–3.4, 4.1–4.3]. Discuss group completion and spectrification of additive functors and use it to construct Grothendieck–Witt spectra from Grothendieck–Witt spaces. By applying Grothendieck–Witt to metabolic Poincaré-Verdier sequence deduce the Bott–Genauer sequence and Karoubi's fundamental theorem.
(9) *L-theory spectrum and the fundamental fibre square* [CDH$^+$20b, §3.5–3.6, §4.4, 4.6]. Discuss bordification of additive functors and use it to construct the L-theory spectrum. Deduce the fundamental fibre square.
(10) *Applications to Hermitian K-theory of rings* [CDH$^+$20c, Introduction]. Summarise the results in the third paper of the series.
(11) *Stable moduli spaces of Hermitian forms* [HS21, Introduction]. Summarise the results in this companion paper.

References

[CDH+20a] Baptiste Calmès, Emanuele Dotto, Yonatan Harpaz, Fabian Hebestreit, Markus Land, Kristian Moi, Denis Nardin, Thomas Nikolaus, and Wolfgang Steimle, *Hermitian k-theory for stable ∞-categories i: Foundations*, 2020, arXiv:2009.07223. 1

[CDH+20b] ______, *Hermitian k-theory for stable ∞-categories ii: Cobordism categories and additivity*, 2020, arXiv:2009.07224. 1

[CDH+20c] ______, *Hermitian k-theory for stable ∞-categories iii: Grothendieck-witt groups of rings*, 2020, arXiv:2009.07225. 1

[HS21] Fabian Hebestreit and Wolfgang Steimle, *Stable moduli spaces of hermitian forms*, 2021, arXiv:2103.13911. 1

[NS18] Thomas Nikolaus and Peter Scholze, *On topological cyclic homology*, Acta Math. **221** (2018), no. 2, 203–409. MR 3904731 1

[WW14] Michael S. Weiss and Bruce E. Williams, *Automorphisms of manifolds and algebraic K-theory: Part III*, Mem. Amer. Math. Soc. **231** (2014), no. 1084, vi+110. MR 3235548 1