ERRATUM TO: SOME FINITENESS RESULTS FOR AUTOMORPHISMS GROUPS OF MANIFOLDS

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Abstract. We give some minor corrections to *Some finiteness results for automorphisms groups of manifolds*.

1. What did Sullivan prove?

The proof of Proposition 3.11 of [Kup19] says that

> Sullivan proved that for closed oriented 1-connected N of dimension ≥ 5, $\pi_0(\mathrm{Diff}(N))$ is commensurable with an arithmetic group [Sul77, Theorem 13.3] and Triantafillou generalized this to oriented manifolds with finite fundamental groups [Tri95].

Manuel Krannich and Oscar Randal-Williams have pointed out this is not correct, because Sullivan's and Triantafillou's use of "commensurability" differs from that in this paper (Remark 2.10): what they call "commensurable" we call "differing by finite groups" (Definition 2.9). More precisely, they prove that $\pi_0(\mathrm{Diff}(N))/C$ is arithmetic for some normal finite subgroup C. In fact $\pi_0(\mathrm{Diff}(N))$ need not be commensurable to an arithmetic group, see [KRW20] for an example based on work of Deligne [Del78]. Remark 2.10 of [Kup19] is thus also incorrect.

Because all we need is that $B\pi_0(\mathrm{Diff}(N)) \in \mathsf{HFin}$, Lemma 2.8 suffices to make the proof of Proposition 3.11 go through. Remark 2.10 is not used elsewhere. Thus this correction has no consequences for any other results in the paper.

Remark 1.1. As pointed out in [Ser79, Example 1.2.(8)], $\Gamma = \pi_0(\mathrm{hAut}(N))$ *is* arithmetic. Firstly, [Sul77, Theorem 10.3] says that Γ/C is arithmetic for some normal finite subgroup C. Let $\pi\colon \Gamma \to \Gamma/C$ denote the quotient homomorphism. Secondly, [Sul74, Theorem 3.2] implies that the profinite completion map $\Gamma \to \hat{\Gamma}$ is injective. This is equivalent to Γ being residually finite, so there is a finite group H and a homomorphism $h\colon \Gamma \to H$ which is injective on C. Then $(\pi, h)\colon \Gamma \to \Gamma/C \times H$ is injective with finite index. Now use that arithmetic groups are closed under finite products, as well as passing to finite index subgroups, and that finite groups are arithmetic. These facts can be found in [Ser79, §1.1 and §1.2].

2. Hatcher–Wagoner in low dimensions

In Lemma 5.23 of [Kup19], it is claimed that for oriented closed connected manifolds M with $\pi_1(M)$ of dimension $n \geq 5$, the classifying space of the kernel of $\pi_0(\mathrm{CAT}(M)) \to \pi_0(\widetilde{\mathrm{CAT}}(M))$ lies in HFin. The argument given only works for $n \geq 6$, as the Hatcher–Wagoner

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sequence is not known to be exact in dimension 5 (even though this is claimed in [Hat78], a proof of this statement does not appear in the literature). This has no consequences for any other results in the paper.

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