ERRATUM TO: THREE APPLICATIONS OF DELOOPING TO h-PRINCIPLES

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Abstract. We correct a mistake in *Three applications of delooping to h-principles*, though it does not affect any of the results.

1. Lemma 20 is incorrect

Johannes Ebert has pointed out that Lemma 20 of [Kup18] is incorrect. This lemma said:

Lemma 1.1. *Let $b \in \mathcal{B}_\Psi(S^{n-1})$, and suppose that Ψ satisfies condition (H). Then $\Psi(D^n_+$ rel $b) \neq \varnothing$ if and only if $\Psi(D^n_- \operatorname{rel} b) \neq \varnothing$.*

A counterexample is provided by letting Ψ be the sheaf which assigns to U the set of everywhere non-zero sections of the tangent bundle TU; that is, the space of everywhere non-zero vector fields on U. This provides a counterexample because it is not always possible to extend an everywhere non-zero section of TS^n over a hemisphere D^n_+ to the entire sphere S^n. The easiest obstruction is the non-vanishing of the Euler characteristic.

The mistake in the proof occurs at the very start: the statement of Lemma 22 is equivalent to $\Psi(D^n_+ \operatorname{rel} b) \neq \varnothing$ if and only if $\Psi(D^n_+ \operatorname{rel} b') \neq \varnothing$ *where b' is the boundary condition obtained by applying the diffeomorphism $D^n_+ \cong D^n_-$*. After having passing to Ψ^f, this amounts to modifying the map $S^{n-1} \to \Psi^f(\mathbb{R}^n)$ by applying the map induced on the frame bundle by the bundle isomorphism

$$D^n_+ \times \mathbb{R}^n \longrightarrow TS^n|_{D^n_+} \longrightarrow TS^n|_{D^n_-} \longrightarrow D^n_- \times \mathbb{R}^n,$$

a composition of reflection *and the clutching map of TS^n*. Thus the proof goes through only if the clutching map can be taken to the identity, i.e. $n = 1, 3, 7$.

2. Modifications to the paper

Lemma 20 does not play a major role in the paper; its purpose was to simplify arguments by removing the distinction between boundary conditions $b \in \mathcal{B}_\Psi(S^{n-1})$ which extend over D^n_+ ("right-fillable") and those which extend over D^n_- ("left-fillable"). Indeed, the results of the paper hold without substantial modification.

Firstly, the reader interested in the applications may replace Lemma 20 by the assumption that for Ψ a boundary condition is left-fillable if and only if it is right-fillable. This is satisfied in all examples in which a homotopical h-principle is proved:

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- Lemma 57 concerns spaces of functions with moderate singularities. Here a boundary condition $b \in \mathcal{B}_{\mathcal{F}(-,\mathcal{D})}(S^{n-1})$ is left-fillable if and only if it is underlying continuous map $f_b \colon S^{n-1} \to Z$ is null-homotopic if and only if it is right-fillable.
- Lemma 73 concerns framed functions, and Igusa's argument gives that every boundary condition is both left- and right-fillable.

Secondly, the proofs can easily be modified to remove any reliance on Lemma 20. Here the required changes:

Definition 19: This needs to be replaced with the following definition of a fillable boundary condition: $b \in \mathcal{B}_\Psi(S^{n-1})$ is *fillable* if it extends to D^n_+.

Lemma 20: This needs to be replaced with the statement that if Ψ satisfies condition (H), then if b is fillable and there is a morphism $b \to b'$, b' is also fillable. Thus the fillable boundary components form a collection of path-components of $[\mathsf{C}^\Psi]$.

This is proven by first using the homological h-principle to pass to Ψ^f and then observing that if there is a homotopy from a section $s_0 \in (TS^{n-1} \oplus \epsilon) \times_{\mathrm{GL}_n(\mathbb{R})} \Psi(\mathbb{R}^n)$ to another section s_1, there is also a homotopy of sections from s_1 to s_0.

Proof of Theorem 27: In this proof we use an embedding $\mathbb{R}^n \hookrightarrow M \setminus A$, from which we derive an embedding $S^{n-1} \times \mathbb{R} \hookrightarrow M \setminus A$: by convention, we do so by composing with an embedding $S^{n-1} \times \mathbb{R} \hookrightarrow \mathbb{R}^n$ with the property that each $S^{n-1} \times \{t\}$ bounds a disk in $\mathbb{R}^n$ *to the right*. This guarantees that all boundary conditions which appears are fillable in the above sense.

In any other place where "fillable boundary conditions" are mentioned, no modification is needed.

Remark 2.1. There is no mathematical reason to define a fillable boundary condition to be one which extends to D^n_+ rather than one which extends to D^n_-. This is a matter of convention.

References

[Kup18] Alexander Kupers, *Three applications of delooping to h-principles*, Geometriae Dedicata (2018). 1

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