Math & Stats Learning Centre is extremely special to me for many reasons. Not only did it help me succeed in math as an undergraduate student, it helped model my career quite a bit as well. I got my first taste of being a TA here at MSLC and for that I will forever be thankful, for it helped provide me a sense of clarity on the field I wanted to pursue a career in.

Being offered the role of putting together another edition of the MSLC magazine has been an absolute dream come true. Given that this is the last time I will be with MSLC as a TA, I saw this as a perfect way to thank MSLC for everything they have done for me over the years. This is a very bittersweet moment, saying goodbye to people and places you come to love and respect is never an easy thing to do. What gives me solace is the group of amazing TA’s and facilitators that continue to work tirelessly to help students find their way.

One piece of advice I want to leave you guys with is—Realize your dreams and pursue them, for your dreams are who you are and you are what your dreams are.

Manaal Hussain
M.Ed. Higher Education
Exploring the beauty of Mathematics through the Euler Formula

By: Zohreh Shahbazi
Associate Professor, Teaching Stream CMS (Mathematics) / Math and Stats Learning Centre Coordinator, CTL

A student once asked me what we mean by the beauty of mathematics, since like most undergraduates, she found doing mathematics taxing and difficult. This gave me a pause, and caused me to think about beauty in general; it’s hard to put into words why I find mathematics so beautiful.

We all understand beauty at face value, even if we can’t describe it. A beautiful object or even a beautiful or elegant concept can fill us with intense emotion and satisfaction. A flower, a birdsong, a pleasant fragrance, a clear, starry sky at midnight... It’s simple enough to think of examples of beauty, but what is beauty itself? They say beauty is in the eye of the beholder, but my own concept of beauty is encapsulated by perfection, simplicity, harmony and symmetry between all parts of a whole system.

Mathematics has all of these things. To help understand what I mean, consider one of the most beautiful mathematical equations, the Euler formula:

\[ e^{i\pi} + 1 = 0 \]

So what exactly is so beautiful about this? This simple formula combines five important numbers \((0, 1, e, i, \pi)\) by using the fundamental operation of addition and the concept of equality. Euler formula summarizes our knowledge about the structure of the number system in a short format. To understand the importance of the Euler formula, let me briefly mention the development of the number system. We use numbers to model the nature around us, and to describe our human social needs. “One” is the most important number—it is the building block of all other numbers. We can obtain all other natural numbers by adding one to itself multiple times. Natural numbers are useful for counting. There are some quantities that we cannot model by natural numbers. By adding zero and negative numbers to the set of natural numbers, we obtain the set of integers. Such extension enables us to describe concepts such as below zero temperatures or deficit values. Next, we construct rational numbers as ratios of integers as long as the denominator is not zero. The set of rational numbers is useful to describe concepts such as half of the capacity of a container. However, there are still some geometric lengths that could not be described by rational numbers, for instance the diagonal of a square. By including numbers like \(\sqrt{2}\) in the collection of numbers, we obtain the set of real numbers.

Numbers such as \(\sqrt{2}\), with a distinction that their decimal expansions are not terminating nor repeating, are called irrational numbers. Among all known irrational numbers, there are two fascinating numbers that appear in the heart of almost all mathematical discoveries. These two numbers are Euler number “e” and the number “\(\pi\)”. Below we can see some of the digits of the decimal expansions of e and \(\pi\):

\[ \pi = 3.1415926536... \]
\[ e = 2.71828182845904523536... \]
Remember that the irrational number $\pi$, is the constant ratio of the perimeter of any circle to its diameter. This famous irrational number is useful for studying phenomena with periodic behavior, such as planets orbiting around the Sun. The irrational number “$e$” was discovered by the Swiss mathematician Leonard Euler in 1727. It is called “$e$” to show that it is originated from the study of “exponential expressions.” It turned out that many calculus formulas could be simplified greatly by exponential functions written in base ‘$e$’. Note that the operations defined on the set of real numbers are addition, subtraction, multiplication and division. With these operations we are not able to find any number to be equal to the square root of $-1$. Once again, we allow ourselves to extend and expand by defining a new number “$i$” to act as the square root of $-1$. That is, $i \times i = -1$. This new invention is quite useful in studying circuits in physics, and we call this number a complex number. Of course, we define operations on complex numbers such that they are consistent with the operations defined on all real numbers.

Numbers are invention of human’s mind to model the nature around us. Different types of numbers were invented to meet our specific needs. The Euler formula summarizes the connection between five of the most important numbers:

$$0, 1, e, i, \pi$$

in a seemingly simple equation. It may not have the impact of a rippling sunset on the beach, though I cannot help but find the elegant simplicity of these numbers together staggeringly beautiful—and hopefully, so can you.
Answering a research question using statistical inference involves setting up a null hypothesis, which describes what you think or believe about a characteristic of a population. Statistical inference refers to the method of examining a population by analyzing a smaller, random sample of that population. To answer this question, we use what we call the p-value: assuming the null hypothesis is indeed true, what is the probability that the sample you collected actually came from the population that the null hypothesis describes? In Statistics, we refer to this resulting probability value as a P-value.

However, there is a pitfall with P-values that many people don’t ever hear about. For one, the concept of “statistical significance” is sensitive to both sampling error and how you set up your research question. Statistical significance refers to the degree to which a researcher can be confident that their results occurred because of the effect being tested, rather than by chance. Sampling errors occur when sampled tested in the experiment does not adequately represent the total population. The P-value is a portion of the area underlying a probability distribution, which refers to the probabilities associated with each possible outcome of the experiment. Since the total area under the probability distribution is always 100%, the smallest P-value that you can observe would be 0. The higher the P-value, the more likely it is that your null hypothesis actually describes the overall population from which you took your sample. The smaller the P-value, the more likely it is that your sample did not come from that population so with tiny P-values, you reject the null hypothesis in favour of the alternative.

Lots of people think that there is something magical about a P-value of 0.05. What this value indicates is that the researcher must reject their null hypothesis for 1 out of every 20 samples collected (1/20 = 0.05), when in fact it’s an accurate representation of the population. Let’s assume that I have 25 students in my class, and I ask each student to collect a sample size of 30 other students’ CGPA’s (Cumulative Grade Point Average); according to the Central Limit Theorem, a sample size of 30 is considered “large enough” for statistical significance. Based on the mean cumulative CGPA from their samples, each student performs a hypothesis test, comparing their own sample’s mean to the University’s published average CGPA of 3.2 (a one sample t-test). As their alternative hypothesis, they estimate that the real population’s cumulative CGPA is less than 3.2.

Every time we do this, at least one or two students reject the null hypothesis, which is expected with a P value of 0.05, indicating that 1 out of every 20 samples must be rejected! As researchers, we tend to be attached to our theories, and sometimes we want them to be correct, even if they aren’t. We are biased towards “statistically significant” findings and thus are more inclined to publish them over statistically insignificant findings. Replication particularly by multiple independent researchers can help control these biases. In early 2015, the editors of an academic journal, “Basic and Applied Social Psychology” made the decision to ban P-values from any research results submitted for publication. Their editorial stated that “the p < 0.05 bar is too easy to pass and sometimes serves as an excuse for lower quality research” - I can’t disagree, since an inference test at this level even within one of my 25-students classes will usually result in at least one student incorrectly rejecting the null hypothesis. If the standard for P-values is this low, it is easier for
researchers to advance incorrect theories by handpicking their data and only publishing findings that support their hypotheses.

Immediately after this announcement was published, a few academics chimed in on the debate (the full article can be found at http://www.statslife.org.uk/opinion/2114-journals-s-ban-on-null-hypothesis-significance-testing-reactions-from-the-statistical-arena; I selected two representative opinions):

“I share the editor’s concerns that inferential statistical methods are open to misuse and misinterpretation, but do not feel that a blanket ban on any particular inferential method is the most constructive response”.

- Peter Diggle, RSS president and professor of biostatistics, epidemiology and population health at Lancaster University

“One big problem is that p is a single value, which suggests certainty, whereas the extent of a confidence interval makes uncertainty salient. On replication, P-Values bounce around amazingly – the ‘dance of P-Values’. Confidence intervals also bounce, but the extent of any interval gives a reasonable idea of the amount of bouncing. In stark contrast, a single P-Value gives almost no idea how much p is likely to bounce on replication.”

- Geoff Cumming, Emeritus professor at the school of Psychological Science at La Trobe University

As indicated by Geoff Cumming, P-Values are very sensitive to sampling errors. So is there an alternative for using P-Values? Although P-values have their pitfalls, I do not think we should give up on P-Values entirely. They play an important role in helping us determine whether there are differences between values measured from multiple groups. However, we need to remember that conducting and communicating research is a complex storytelling endeavour: rather than telling a story using just one piece of evidence (the P-Value) we should be telling our story using multiple methods. But what other pieces of evidence should we use, and how can we use them?

- Construct confidence intervals around your estimates.
- Perform power analysis to ensure that your sample sizes are large enough to detect an effect
- Pay more attention to methods that focus on determining effect sizes
- Supplement your study with meta-analysis (that is, specifically look for the results other researchers have come up with and see how your results compare)
- Share your results, even if they are not statistically significant
- Make your data open and accessible, and provide enough information that other researchers can replicate your study

This last element makes it possible for other researchers to replicate your studies and verify your findings, or to combine new data with the data we have collected (called “Open Science”). Collecting more data, and replicating studies, can be costly. As a result, we don’t typically do it ourselves, and we don’t make it easy for other investigators to do either. If we truly wish to move mathematical research and inquiry forward, this needs to change.

“Significant” – an XKCD cartoon by Randal Munroe (http://xkcd.com/882/)

This is a part of the cartoon, please follow the link to look through it.
Numbers can be scary. When you step inside an elevator to go up to the fourteenth floor of a building, you know you aren’t really going to the fourteenth floor. You’re going to the thirteenth floor, but we can’t call it that; the number thirteen is considered taboo or unlucky across many cultures, including our own. People pick lottery numbers or sports jerseys based on the perceived significance (in this case, luck) of certain numbers over others. Some even apply deep spiritual significance to certain numbers (such as 666 or 777). Like many of the students who come to the Math and Statistics Learning Centre, people have feared numbers (both specific numbers, and numbers generally) dating back to the ancient world. These days we have fancy terms for this: numerophobia or arithmophobia, the irrational fear of numbers.
A lot of students think of the more advanced branches of mathematics (calculus for instance) as existing in an entirely different category from basic arithmetic, slope, and the other types of math they learned in high school; but like anything else, advanced mathematics aren’t magical theoretical singularities, but rather advanced applications of the basics. But there’s one particular form of numerophobia or arithmophobia that various societies had to overcome in order to allow these advanced branches of mathematics to exist in the first place: Oudenophobia, or fear of the number zero.

It would seem that numbers have had this kind of significance for nearly as long as we’ve been writing them down. Most early written languages did not make use of a place holder zero like with our base-10 number system; for example, in the number 100, we use two zeroes to signify that there are no additional numbers beyond one hundred. In most early written languages, 100, 1000, etc. would not make use of the number zero, but would rather have entirely unique symbols to represent them. The first evidence we have of the use of the number zero is in ancient Egypt, around 1740 BCE. By the time of the early Christian church, scholars and theologians had begun debating not only whether or not the number zero should exist as a concept, but also whether or not it was considered in some way blasphemous.

Without zero, our society couldn’t function (pun intended!). Without zero, we wouldn’t have calculus, which we need in order for, well, pretty much everything... That’s right, without a little symbol that represents nothing, our computers wouldn’t run; without zero, engineers wouldn’t be able to make the calculations necessary to ensure that our cars are safe to drive, and accountants wouldn’t be able to balance our budgets.

So, if you find math and numbers intimidating, you’re not alone. We all come from a long lineage of numerophobes, arithmophobes and oudenophobes. At the Math and Stats Learning Centre, we’re here to help. Whether you’re an aspiring mathematician or not, we want to show you that math and numbers don’t have to be scary—no, we need to show you that math and numbers don’t have to be scary. Like with zero, we have to shift the perception of mathematics away from fear and stigma if we ever hope to pave the way for the next big breakthrough—the next branch of mathematics that revolutionizes the way we build machines and program computers, so that we can build safer vehicles, less frustrating computer programs and whatever else. And it starts with you—so come see us, and we’ll help you to fear one less number than you do now: your GPA!
The MSLC supported me through my first year calculus course. I have always had a lot of trouble with math, and after taking a gap year I was even more nervous about taking university-level calculus. I discovered the MSLC and am extremely grateful for their guidance. The MSLC was always there to help with web homework, explaining concepts and exam preparation. There was always someone there to answer my questions and walk me through a problem. The MSLC is a great resource available on campus that I wouldn’t hesitate to recommend to any mathematics student.

The MSLC was a huge benefit to me ever since I started going last semester. Statistics was the one course I was dreading to do ever since the start of my program. I nearly failed grade 11 functions and I did not take any grade 12 math courses. Anything related to math has been an absolute nightmare for me. After hearing one of the people from the MSLC make an announcement in my class at the start of the term, I thought I might give it a try, to see if I can pull off a halfway decent mark. It was not easy to say the least at first, but it slowly started getting much easier. The tutors really helped to explain the concepts in ways that I could easily understand.

By the end of the semester I was nearly a tutor myself at the stats centre. I routinely came and formed study groups with other students whenever the centre got really busy. I was really quite confident in my ability to handle stats, and I surprisingly started enjoying statistics far more than I ever thought I would. I ended with a mark I am quite proud of in the course, and I am excited to take my advanced statistics course next semester.

It was really really helpful having a one on one tutor for some concepts that I did not understand during the lecture. As a result, I understood those difficult topics even better than I would have understood in the lecture.

I struggled a lot with Calculus and after seeing my mark on the midterm I figured that I needed some help. I started attending group tutoring sessions and they helped me go over the questions so that I understood what was going on (I found the lectures really really confusing). This really helped me to do better. My mark on the last assignment I did pre-tutoring was 40%, that jumped to 100% after two weeks of tutoring.
Bar graph highlighting student usage of MSLC for various courses

Pie chart demonstrating student satisfaction with MSLC
During several of our regular TA drop-in sessions, we anonymously interviewed students in the Centre about their experiences getting help at the MSLC. We interviewed students from a wide variety of programs, from first years to mature students. When we asked the students why they’d come to the MSLC for extra help, we got a variety of responses:

“Because I failed a midterm. Straight up.”

“When I get stuck on questions, sometimes they aren’t necessarily hard but I need a new approach to attack the question. Having someone to talk to helps with that.”

“Math was something I hadn’t done for a long time. I didn’t even know how to do the functions on the calculator.”

“Sometimes I ask questions and they trigger more questions and I can’t answer them.”

“A lot of questions that don’t have answers—I want to see if I’m doing them right. A lot of questions I don’t know how to approach, so I ask for help. I also want to learn some of the skills that the TA’s have by seeing how they do the questions.”

Next we asked students if coming to the MSLC had improved their individual study habits, or given them transferable skills useful outside of math courses. Every single student we interviewed told us that they had recommended the MSLC to their friends:

“If it wasn’t for the MSLC I probably wouldn’t be studying as much. I’d probably just cram.”

“I’ve now gone to go get help in other areas like this.”

“The MSLC gave me a new perspective on what directions I could go in solving problems.”

“It helps me understand so that I can work on my own.”

“Going over questions with the TA’s has really helped me to grasp how we’re supposed to do the questions in the first place. It’s a different thinking style. The TA’s already know what they’re doing, but I don’t. When I first started coming to the MSLC, the TA’s more or less did the questions and I just tried to keep up and understand what they were doing. Now I can do the questions and I just ask for occasional help. It changed the way I think about the subject.”

“I learned how to teach or explain things to people. A lot of the time there’s not enough staff, so the students help each other out.”

“Yeah, if a friend asks me for help I employ the same sorts of techniques used by the TAs.”

“I learned how to use my calculator [laughs]. I couldn’t have passed that course without the MSLC. They were always there for me and they were really patient. They were very thorough and they knew their stuff really, really well.”

“Problem solving. It used to be hard for me to start solving a problem, but the MSLC helped me to learn how to stay calm and break a problem down bit by bit.”

“Not giving up.”

“It’s helped me communicate what the problem is and explaining to someone, ‘this is what I need help with.’”

“It’s really increased my willingness to ask people for help. When I first came to U of T, I tried to do everything on my own. Now I’m far more likely to ask someone for help when I get stuck.”

Lastly, we asked the students whether or not they like math or stats:

“I do now. It’s because I’m getting a good mark.”

“Yeah, I like it. I didn’t realize I was so lame in it until I took stats [laughs].”

“[Laughs] What do you mean, like?”
Centre of Teaching & Learning at UTSC recently developed 12 online modules to provide learning support for students taking calculus courses. Modules 1-8 cover foundational concepts and modules 9-12 cover more advanced concepts. These modules can be used as self-directed learning support tools based on the needs of students and can also be incorporated into the curricula of calculus courses. Various introductory courses at UTSC allow the use of these modules as a self-learn tool for their students. These modules act as a means to review a specific concept and try out various tests to check your understanding.

**Animations:** We have developed 2-3 minute animations for each of the 12 modules (for a total of 39 minutes). The main purpose of the animations is to introduce the topics in an innovative way and engage the learner. The animations may also help students to visualize some of the more difficult topics. Our goal was to create a learning tool that would help to engage students and change their negative attitudes toward learning mathematics and make it a fun-filled experience.

**Instructional Videos:** Each module includes 2-5 videos which model approaches to problem solving in mathematics. There are around 10 hours of instructional videos in total. The main purpose of these videos is to engage students to learn mathematical concepts in a simple, natural, and constructive manner.

For accessibility purposes, the animations and videos are provided with audio and descriptive video transcripts.

**Additional Resources:** For modules 1-8, additional problem sets, diagnostic tests, and assessment tests are provided along with detailed solutions (a total of about 80 pages). For modules 9-12, which focus on more advanced calculus concepts, we have developed a 40-page handbook explaining selected topics with carefully chosen examples, along with additional practice questions and answers. These resources will help students gather a better understanding of the concepts and improve their problem solving skills.

**Implementation:** These modules are used as a part of UTSC’s Online Mathematics Preparedness Course, which is offered each semester. Students who receive 65% or higher in this non-credit course can use it to fulfill high school mathematics prerequisites that their programs may require. If you are a student missing the Math/Calculus prerequisites to take a course here at UTSC, feel free to contact Centre of Teaching & Learning for registration details.
At the MSLC, we’re interested in multidisciplinary perspectives, and making use of different approaches to teaching and learning. For this issue, we interviewed UTSC’s own Professor Andrew Westoll, a primatologist turned creative-writer.

After completing his BSc. in biology, Professor Westoll studied primates in the jungles of Suriname, a small, Dutch-speaking country in South America. After returning to Canada, Professor Westoll obtained an MFA in Creative Writing from the University of British Columbia and published his first book, The Riverbones, a memoir detailing his travels in Suriname. His second book, The Chimps Of Fauna Sanctuary: A Canadian Story Of Resilience And Recovery, won the 2012 Charles Taylor Prize for Literary Non-Fiction. In this interview, we asked Professor Westoll about interdisciplinary approaches to teaching, the differences between the sciences and the humanities, the effect of fatherhood on his teaching style, and more.

**MSLC: How has your background in science affected your approach to teaching English and writing?**

**Andrew Westoll:** My science background has led me to formulate a few useful frameworks that I can use to discuss various aspects of creative writing. Without my training in biology, I probably wouldn’t be so keen to find overarching organizing principles to help lead classroom activities. I’m thinking Story & Style in my CNF [creative non-fiction] classes, in particular.

**MSLC: Do you think the way the sciences and humanities are taught at the undergraduate level is too regimented?**

**Andrew Westoll:** Yes, and I think it’s underway as I write this. Since I started ENGB52 I’ve heard from lots of my colleagues, from across the scholarly spectrum, who are interested in either sending their students my way or collaborating in some way. It’s a very exciting time to be on UTSC campus for this very reason!

**MSLC: Do you think there could be more of an effort at UTSC to bring different disciplines together (like your ENGB52 course Literature and Science)?**

**Andrew Westoll:** Ha! I’m not sure it’s possible to be a Renaissance person anymore. I think proving your
abilities and pursuing your curiosities in one distinct area is always useful. Some of us are just lucky enough to be able to do this in multiple arenas. But these arenas are always connected, as I’ve already suggested. I think the best way forward for young ambitious people is for them to nurture their creativity and curiosity, no matter where that may lead.

MSLC: Was being a writer your dream all along, or were you previously trying to juggle two passions at once (while academically pursuing the arguably more employable one)?

Andrew Westoll: I really wanted to study animals in the wild. Then I did that, and I realized what I really wanted was to experience the wild. Then I did that, and I realized what I really wanted was to write about those experiences. Then I did that, and... you get the point. I just went after what I wanted, was fortunate and privileged enough to catch some breaks along the way, and one thing led to the next. Writing was always there in the background, though, yes indeed!

MSLC: Do you have any advice for students or recent graduates looking to switch gears like you’ve done? Should students unashamedly pursue their passions, pick something safe, or somewhere in between?

Andrew Westoll: I hope I’ve already answered this, in a way. As for safety and security, the most important lesson is that everyone has their own tolerance for instability and risk. You have to find your own. I walked that line for a long time, waking many a morning asking myself, “What the heck am I doing?” I had a good stomach for insecurity when I was younger, now less so. But during that time (in my twenties) I took full advantage of my comfort with risk. That’s always a good idea, if you discover you’re willing and able. Just go for it. Your body will end up telling you when you’ve gone too far. That’s another lesson: It won’t be your mind that speaks up; it’ll be your body.

MSLC: Has becoming a father affected your teaching philosophy?

Andrew Westoll: Great question. I’m not sure yet, although there are some obvious connections between being a father and a teacher. I think I’ll find out over the coming years. A colleague told me recently that now I have a child, I won’t be able to put as much of myself into my teaching. I totally understand where this comment was coming from, and that my colleague was right, but that worries me a bit. I don’t want one part of my life to cannibalize the other. I’d prefer to find a way for them to feed each other. Maybe I’ll be singing another tune come the fall, when I’m back teaching full-time!
Worried about your future?
Take a look at your MSLC Horoscope!

AQUARIUS (Jan. 20th - Feb. 18th)
The coming months will be a roller coaster ride of good and bad fortune, with your life following the trend of a sine function. Enjoy the maximums while they last, and when the minimums come, remember that your life resembles a periodic function—the good fortune will return again soon!

PISCES (Feb. 19th - March 20th)
Life is like a random variable! You never know what it will bring you! Not only that, but it is a continuous random variable, and so you must always keep in mind that it can take on absolutely any value. Embrace this randomness as unexpected events arise in your life. Even though your world will soon have an enormous standard deviation, stay calm until things hover around the mean once more!

ARIES (March 21st - April 19th)
Despite fears about results from a recent examination, you have no reason to worry, for your z-score is 2.89! Congratulations! All your hard work has paid off, and you will soon be able to bask in the glory of the 99.81st percentile! Enjoy the feeling of pride you get upon receiving your results—you deserve it!

TAURUS (April 20th - May 20th)
Lately you have been feeling overwhelmed with life, like a concave down function whose slope just keeps getting more and more negative. Fortunately, the coming weeks will bring a local minimum to your problems, and soon you will have a positive slope once more! Just hang on until your concavity changes—you will be feeling on top of the world in no time!

GEMINI (May 21st - June 20th)
Often we get so caught up in life that we forget to look back at how amazing we truly are. You too have gotten engulfed by life’s stresses, and now are feeling inadequate compared to your peers. Soon, though, you will be recognized for your efforts. Take a moment and integrate the function of your life—you’ll be amazed at the size of the area that lies beneath the curve!

CANCER (June 21st - July 22nd)
Stop making linear regression equations out of your life and trying to predict every outcome! Lately, you have gotten too caught up in trying to make sure everything is lined up perfectly, and in predicting the exact value of each result in your life. Relax! Don’t let residuals scare you! Sometimes life is meant to deviate from the norm and it is best just to take the random error as it comes.

By: Olivia Renee
Statistics Facilitator
**LEO (July 23rd - Aug. 22nd)**

Great fortune will soon come to you, but only if you keep putting in the right amount of effort! Right now you are at the minimum of the curve, and perhaps feeling slightly down about life, but keep moving forward! Increasing values of x will enable you to approach a vertical asymptote, ultimately leading you to a value of $+\infty$!

**VIRGO (Aug. 23rd - Sept. 22nd)**

Lately you have been comparing your options, and wondering whether the path of hard work and dedication is truly worth it. Questions about whether taking the harder route will truly pay off have caused a lot of worry for you. Take a moment and do a statistical analysis of your life, though. Without a doubt, a t-test comparing the mean value of satisfaction derived from hard work versus easy results at $\alpha=0.01$ will yield highly significant results! You can, at the 99% level of confidence, reject $H_0$, and realize that sometimes the best things in life require commitment and perseverance!

**LIBRA (Sept. 23rd - Oct. 21st)**

You have been working extremely hard lately, trying to achieve a very important goal in your life. However, things over the past few months have not made it appear as though any progress towards this goal is being made, making you feel slightly discouraged. Soon, though, everything will become clear, and if you create a scatterplot of your hard work versus achievements, you will find Pearson’s $r$ will be equal to 1!

**SCORPIO (Oct. 22nd - Nov. 21st)**

We all want our lives to follow a normal distribution, with stability, and a mean not pulled up or down by unforeseen outliers. Don’t be afraid of skewness, though! While the shape and spread of your life may be non-normal, very large outliers are coming your way! Incredible outliers that will pull your mean up substantially, and open new doors that you never even knew existed!

**SAGITTARIUS (Nov. 22nd - Dec. 21st)**

The world is a confusing place, and sometimes you wonder whether the path you are on will lead to anything of use. Fear not! Your life is like a function with a domain and range of $(-\infty, +\infty)$! Absolutely anything you dream of can be achieved, if only you try inputting enough values of x! Just keep persevering, never give up, and soon you will end up with the value of y that you have always hoped for!

**CAPRICORN (Dec. 22nd - Jan. 19th)**

Even though you have experienced trouble in a certain area of your life in the preceding months, things will get better very soon! Despite feeling like the function of your life has had a slope of zero, take a moment to differentiate, and you will see that things really do have a very positive slope! Stop, take a break, and determine the value of x you are at right now. Using some time to slow down and figure out the slope at this point will enable you to see that things have been positive, rather than zero, all along!
1. Given non-negative real numbers \(a, b, c\). Prove that:

\[
(a^2 + b^2 + c^2)^2 \geq 4(a-b)(b-c)(c-a)(a+b+c).
\]

**Solutions:** Let

\[
f(a, b, c) = (a^2 + b^2 + c^2)^2 - 4(a-b)(b-c)(c-a)(a+b+c)
\]

We need to show \(f(a, b, c) \geq 0\). Since \(f(a, b, c) = f(b, c, a) = f(c, b, a)\), without loss of generality, we can assume \(c = \min(a, b, c)\).

If \(a \geq b \geq c\), then \(f(a, b, c) \geq 0\).

Now consider \(b > a \geq c\):

\[
f(a, b, c) - f(a, b, 0) = (a^2 + b^2 + c^2)^2 - (a^2 + b^2)^2 - 4(a-b)((b-c)(c-a)(a + b + c) + ab(a + b))
\]

\[
= (a^2 + b^2 + c^2)^2 - (a^2 + b^2)^2 + 4c(b-a)(a^2 + b^2 + ab - c^2) \geq 0
\]

Hence, \(f(a, b, c) \geq f(a, b, 0)\). We also have,

\[
f(a, b, 0) = (a^2 + b^2)^2 + 4ab(a^2 - b^2)
\]

\[
= a^4 + 4a^2b^2 + b^4 + 4a^3b - 2a^2b^2 - 4ab^3
\]

\[
= (a^2 + 2ab - b^2)^2 \geq 0.
\]

Thus, we have \(f(a, b, c) \geq 0\).

Equality occurs when \(c = 0\) and \(a^2 + 2ab - b^2 = 0\). Solving the second condition for \(a\), we have

\[
a = (\sqrt{2} - 1)b \geq 0, \quad a = -(\sqrt{2} + 1)b \leq 0.
\]

Since \(a, b, c\) are non-negative, equality occurs when:

\[
(a, b, c) = \{(\sqrt{2} - 1)t, t, 0); (t, 0, (\sqrt{2} - 1)t); (0, (\sqrt{2} - 1)t, t) \mid t \in \mathbb{R}^+\}
\]