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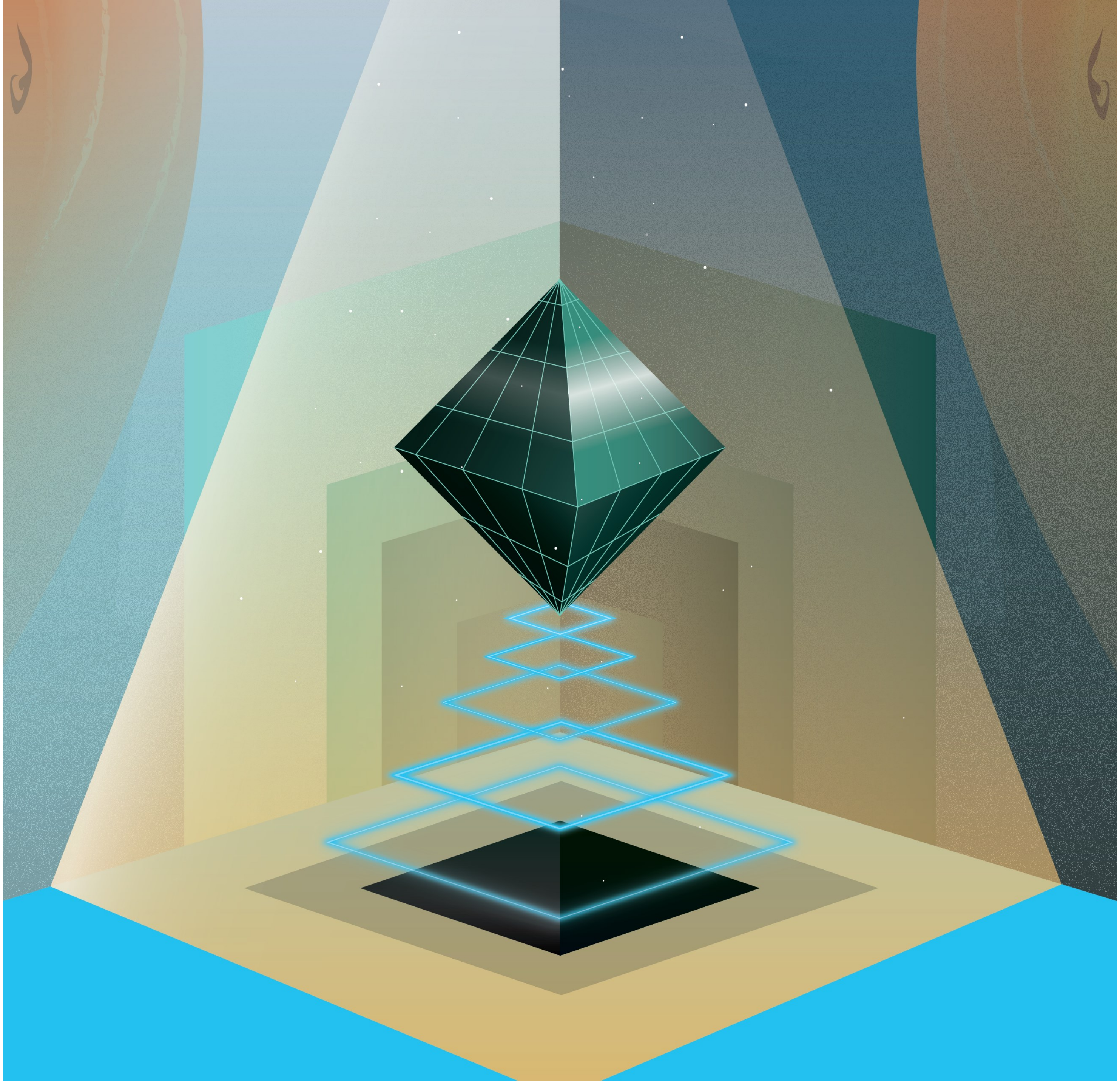
THE SYMMETRY ISSUE



NOTE FROM THE EDITOR

Mslc 2018

Every time I sit down to write this column, I realize how quickly time flies. It seems like a few days ago that I wrote a similar column for a previous issue. I remember being taught that time is relative, and that it is all about how you perceive it--the clock never ticks any faster or slower than usual. This is a concept that I never really understood until recently.

Whenever I think of symmetry, I think of beauty. This beauty often lies in the simplest things and doesn't have to do with the physical appearances of people or things. We took it upon ourselves to highlight beauty and perfection within the field of mathematics in this edition of our magazine.

There's beauty in being the first ever female mathematician to win the prestigious Fields Medal for her work (pg. 3). There's immense beauty in being able to draw and understand connections between fields. Such connections help us to use mathematics and statistics to make real life more meaningful. Mathematical and statistical modelling help us to create models for understanding real life phenomena such as the basic building blocks of life--DNA (pg. 9)--or the Toronto Subway (pg. 7). Finally, we love and appreciate the beauty in being able to challenge stereotypes and empower individuals to push boundaries, from an undergraduate student proving a formula left unsolved by legends (pg. 12) to encouraging girls to pursue and imagine a career in STEM fields (pg. 5).

Till next time,
Manaal
B.Ed., M.Ed.
UTSC Alumni and Staf

INSIDE THIS ISSUE

MATHEMATICS: A LAND OF HUMAN UNDERSTANDING

Prof. Zohreh Shahbazi

Mathematics consists of many subfields. Each subfield aims to solve specific problems with specific approaches and methods generally not used in other subfields. However, the solutions to some problems can only be found through utilizing methods from two or more subfields that may seem unrelated on the surface. In this article I plan to explain two of the most remarkable interdisciplinary discoveries in the history of mathematics.

350 years ago the French mathematician Pierre Fermat proposed a problem related to the natural solutions of the equation $x^n+y^n=z^n$. He claimed that he had discovered an elegant proof of the fact that this equation has no natural solution for n greater than 2--but he couldn't record all of the steps of the solution because he ran out of space. This problem is what we refer to as Fermat's Last Theorem. Many attempts were made to prove Fermat's claim, all of which failed until 1994, when Andrew Wiles and Richard Taylor provided a complete proof for Fermat's Last Theorem. They did so by bridging two seemingly disparate subfields of mathematics: number theory and harmonic analysis. They were able to define a one-to-one association between modular forms from harmonic analysis and solutions to cubic equations moduli a prime number from number theory.

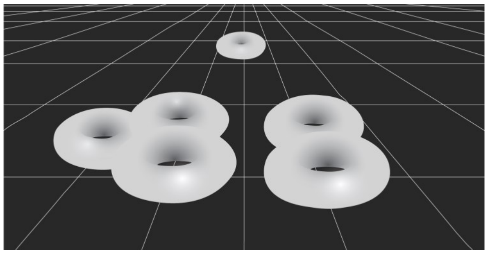
A second example is the work of Maryam Mirzkhani, an Iranian born mathematician who received the prestigious Fields Medal for her work. She was the first ever female mathematician to receive this award. Below I will try to provide an overview of some of her results by first introducing concepts such as Riemann Surfaces, hyperbolic planes, genus of a surface, simple loops and her counting bound for the number of simple loops on Riemann surfaces.

Complex curves or Riemann surfaces are curly surfaces which we use to perform complex analysis. Appropriate metrics allow us to measure the angles, length and area on those surfaces. The Cornell University mathematician Daina Taimina constructed a physical model for an important class of Riemann surfaces called hyperbolic planes by crocheting and adding stitches with a constant ratio to each row. In nature, lettuce leaves and the frills of sea slugs resemble hyperbolic planes. On the side is an image of a crochet model of a hyperbolic plane.

It is interesting to note that the usual rules of classical (Euclidean) geometry no longer apply on a hyperbolic plane. For instance, straight lines no longer follow the usual parallel axiom. This means that from a given point outside of a line, there are infinitely many possible parallel lines, instead of a single parallel line on the Euclidean plane. As a result, the sum of the angles of a given triangle on a hyperbolic plane is always less than 180 degrees.

Euclidean geometry is useful on a smaller scale, but on a larger scale--for example, such as when we travel around the world--we need to do the calculations on a curly surface (like the earth, which looks like a sphere from farther away). On the scale of the universe we might need other forms of geometry such as that of hyperbolic surfaces. Thus the study of Riemann surfaces and their properties can help us to discover important hidden truths about our world.

The number of holes in a Riemann surface is called the genus of the surface, and it turns out that all Riemann surfaces with the same genus have the same geometry. So, the genus of a surface plays a crucial role in how we understand its properties. Below is an image of the surfaces of genus 1, 2 and 3.



A simple loop on a Riemann surface is the shortest possible path between its two points that does not intersect with itself. In 2008, Maryam Mirzkhani proved that the number of simple loops whose length is less than a bound *K* on a Riemann Surface with genus *g* is of the order of K^{6g-6} .

Maryam's work brings together several mathematical disciplines, including hyperbolic geometry, complex analysis, topology, string theory and dynamical systems. Diana Taimina once said that mathematics is an art of human understanding, and Maryam's work demonstrates this beautifully [5].

It is my hope that by discovering more connections between the diverse subfields of mathematics, we will reveal a united mathematical land, one defined by the symmetry of nature, rather than one cobbled together from disparate pieces.

References:

1) David D. Ben-Zvi, "Moduli Spaces" https://www.ma.utexas.edu/users/benzvi/math/pcm0178.pdf

2) "The Work of Maryam Mirzakhani", 2014. https://www.mathunion.org/fileadmin/IMU/Prizes/Fields/2014/news_release_mirzakhani.pdf 3

3) Edward Frenkel, "*Love & Math: The Hidden Reality*", 2013.

4) Zohreh Shahbazi, "*Exploring the Beauty of Mathematics through the Euler Formula*", MSLC Magazine, Summer 2016.

5) Daina Taimina, "Crocheting Adventures with Hyperbolic Planes", 2009.

WOMEN IN STEM

Math in Motion

Prof. Sophie Chrysostomou

We see it all the time – a young girl who excels at math but decides not to pursue science, technology, engineering or math (STEM) after she leaves high school. It's disheartening to any woman educator to see so many talented young women close the door on careers that can be truly fulfilling.

A Natural Sciences and Engineering Research Council of Canada study found that despite women greatly outnumbering men on university campuses, they only make up 39 per cent of undergrads in math and physical science, and only 17 per cent of undergrads in engineering and computer science.

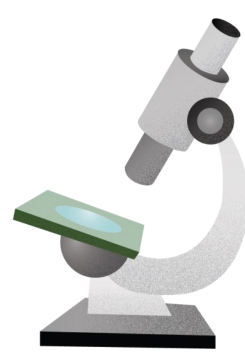
A talent for mathematics doesn't seem to be a factor. One Statistics Canada report found that only 22 per cent of girls who had math marks in the 80 to 89 per cent range in high school chose STEM programs compared to 52 per cent of boys. Incredibly, only 41 per cent of girls with marks in the 90 to 100 per cent range in high school chose STEM programs compared to 61 per cent of boys.

So why is it that so many young talented women avoid pursuing science, technology, engineering and math programs?

Part of the reason may be social. While it ignores the countless contributions made by women mathematicians, computer scientists and engineers around the world, there's also a persistent stereotype that math is somehow considered masculine.

One grassroots initiative to get young women excited and engaged about studying math is an annual event called "Math in Motion…Girls in Gear!" I am very proud to say that an ex-student of my mine, Judy Shanks, spearheaded this conference in 2003 at the Durham District School Board. Judy is a math educator at the DDSB. Needless to say, I have joined her and presented at these conferences. The one-day conference inspires and motivates young girls to pursue mathematics studies through presentations from women professionals explaining how a math education shaped their careers, as well as interactive fun demonstrations and activities involving mathematical principles."

Joined by Carol Miron (a math teacher at Don Mills Collegiate Institute, former classmate of Judy Shanks and former student of mine) and a wonderful team of talented volunteers, we brought this event to UTSC in 2012.



Drawing together women from the academic and corporate worlds as role models, we aim to inspire Grade 9 girls to continue studying math throughout high school and enrol in math-related university programs such as engineering and computer science.

Through the years we have been fortunate enough to have had influential keynote speakers with strong messages. The very leader of our country, Prime Minister Justin Trudeau, sent a video message to the participants stating that "Girls can do anything" and has told the girls to "believe in your dreams because the rest of us are counting on them. All of you are tremendously powerful and our world needs you as leaders."

Last year, “Math in Motion… Girls in Gear!” took place on November 25th in the IC building. We had 101 grade 9 girls participate and 43 female UTSC student-volunteers. We started the day with an eloquent speaker, PwC Partner Lana Paton, and followed with hands on workshops lead by women from the industry and the academic world. During the day the girls had the opportunity to solve puzzles and to engage in a design challenge that had everyone holding their breath! It was a very egg-xciting challenge! Finally we had our keynote speaker who was none other than the honourable Kirsty Duncan, Minister of Science. She was a passionate speaker, and afterwards, the students surrounded her, asking her for autographs and pictures.

It was also a great surprise for all of the participants to see a video clip from the Prime Minister sent just for this conference to relay the message that “Girls Can Do Anything”. The IC building has never been so full of excitement, surprise and … math! There are plenty of pictures that attest to this at mimgig.com.

Regardless of whatever it is that prevents young women from choosing STEM programs, the consequences cannot be ignored. The decision not to pursue STEM programs by young women can have an impact on their earning potential, independence and ability to pursue a stable, meaningful career down the road.

We are hoping that “Math in Motion… Girls in Gear!” will make a difference, but that is not enough. I believe it’s up to all of us – parents, educators and policy-makers – to make sure the support and resources are there for young girls to reach their full potential.

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STUDENTS’ COLUMN

We would like to hear from you!

These columns are available in every issue for your comments, ideas, suggestions, or any topic you would like to share with us regarding the Math & Statistics Learning Centre.

Please send your contributions to math-aid@utsc.utoronto.ca or drop a hard copy off in AC312.
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Introduction:

Toronto's subway network consists of four lines, as shown in Figure 1. The shape of the network is somewhat iconic, with one line going across, and one down and up in a U-shape.

Let's suppose that all I have is a list of the stations on each line. How could I represent the structure of the network, how could I draw it, and how would my image match up to reality? Let's find out.

I will be using the R software for this project. R is software for doing statistics, along with attendant data-science tasks like organizing data. There is no statistics here, but there is a certain amount of data organization that we need to do. I will show the commands I used, in case you know some R and want to follow along. Or, of course, you can just admire the results. There is a pretty picture at the end.

The data:

I entered the station names into a file, one after the other, with NA as a special code to mark the end of one line and the beginning of the next. The top of the data file looks like this:

```
## McCowan
## Scarborough Centre
## Midland
## Ellesmere
## Lawrence East
## Kennedy
## NA
## Kennedy
```

Making a network out of the data:

R comes with add-on packages (as Python does) to provide extra functionality. I will be using these three:

```
library(tidyverse)
library(tidygraph)
library(ggraph)
```

Down to business. Using R, I read this file in as a one-column CSV, which is stored at the URL shown:

```
my_url="http://www.utsc.utoronto.ca/
~butler/subway.txt"
subway=read_csv(my_url,col_names =
"station")
```

To help me figure out which stations are connected to which, I need to make a second column which contains the previous station in the list, which I can do this way:

Figure 1: Toronto's subway network

```
subway = subway %>% mutate
prev_station=lag(station))
subway
```

```
## # A tibble: 83 x 2
##    station          prev_station
##    <chr>            <chr>
##  1 McCowan          <NA>
##  2 Scarborough Centre McCowan
##  3 Midland          Scarborough Centre
##  4 Ellesmere        Midland
##  5 Lawrence East    Ellesmere
##  6 Kennedy          Lawrence East
##  7 <NA>             Kennedy
##  8 Kennedy          <NA>
##  9 Warden           Kennedy
## 10 Victoria Park    Warden
## # ... with 73 more rows
```

and then I need to get rid of the rows with NA in them anywhere, for example to reflect that McCowan station only has one neighbour:

```r
subway = subway %>% filter(!is.na
(station), !is.na(prev_station))
subway
```

```
## # A tibble: 76 x 2
##   station             prev_station
##   <chr>               <chr>
## 1 Scarborough Centre McCowan
## 2 Midland            Scarborough Centre
## 3 Ellesmere          Midland
## 4 Lawrence East      Ellesmere
## 5 Kennedy            Lawrence East
## 6 Warden             Kennedy
## 7 Victoria Park      Warden
## 8 Main Street        Victoria Park
## 9 Woodbine           Main Street
## # ... with 67 more rows
```

The mathematical structure of these data is a graph, with stations as nodes and the rows of the above showing which nodes to connect by an edge. Let's make the above into a so-called `tidygraph` graph, which provides a data structure that includes nodes *and* edges:

```r
subway_g=as_tbl_graph(subway)
subway_g
```

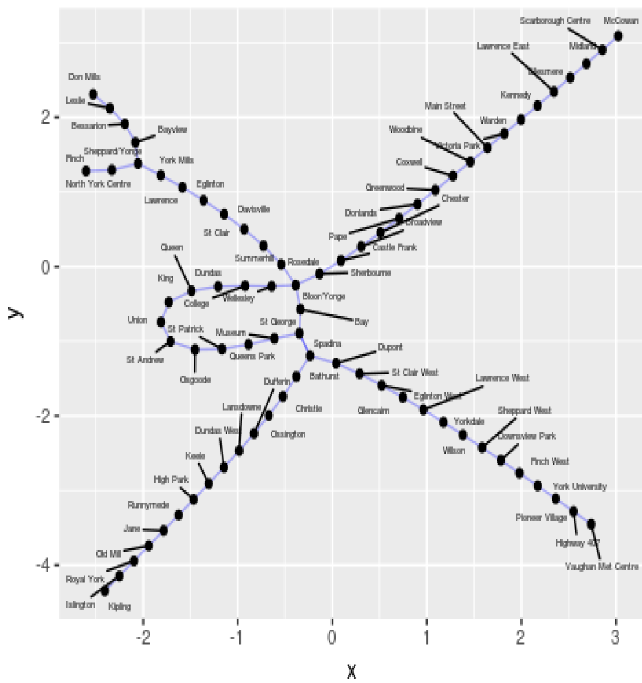
```
## # A tbl_graph: 75 nodes and 76 edges
## #
## # A directed acyclic multigraph with 1 component
## #
## # Node Data: 75 x 1 (active)
##   name
##   <chr>
## 1 Scarborough Centre
## 2 Midland
## 3 Ellesmere
## 4 Lawrence East
## 5 Kennedy
## 6 Warden
## # ... with 69 more rows
## #
## # Edge Data: 76 x 2
##   from    to
##   <int> <int>
## 1     1   74
## 2     2    1
## 3     3    2
## # ... with 73 more rows
```

Drawing the graph:

This is a planar graph (the subway network exists on the surface of the earth, and lines cross over other lines only where there is a station allowing you to change trains there). So it ought to be possible to draw it in two dimensions. But how?

There are different ways to do that. The Kamada-Kawai method tries to place each node a unit distance from its neighbours, and imagines that each edge is a unit-length spring. I like the results it gives:

```r
ggraph(subway_g,layout="kk")+
 geom_edge_link(colour="blue",alpha=0.3)+
 geom_node_point()+
 geom_node_text(aes(label=name),size=1.5,repel=T)
```



This reproduces the structure of the network, with each station connected to its neighbour correctly. But the actual geography is messed up. Some of it is right: McCowan and Kipling are in about the right places, but Finch and Vaughan are definitely not, and the loop appears to go west rather than south and is actually oriented backwards, or would be if it were facing the right way. Also, Don Mills station is supposed to be north of Pape and relatively close to Scarborough Centre. But there is no way for the graph to "know" that, since it is extra information that is not in the knowledge about which station is connected to which. In general, the parts of the subway lines outside of the loop can be drawn in any direction and still maintain the connection information, and the direction of the loop can be reversed (as here). In addition, the stations are not all the same distance apart (Victoria Park to Warden is a vigorous walk).

Getting a geographically accurate map would involve finding the actual locations of the stations (latitude and longitude) by geocoding, and then superimposing the stations and the lines between them on the map. But that is another story.

BRAIDS IN MATH & BIOLOGY

Afiya Vahora & Bhakti Bhatt

Abstract mathematical concepts are developed by studying our observable surroundings. Early mathematicians observed physical structures and relationships in nature and represented them in abstract ways so that it could be applied elsewhere. One such observation was of braids, which are found everywhere: in hairdressing, jewelry, ropes, belts, and many more decorative objects. Braids are studied using the mathematical structure known as the Braid Group.

The Braid Group, denoted by B_n where n is a positive whole number, is a group whose elements are braids with n threads. An example of an element contained in the group B_4 as shown in Figure 1.

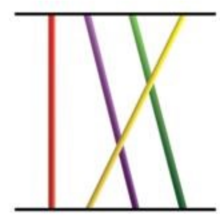


Figure 1

This braid consists of two disks with four points in each where all points from the top disk are connected to the points on the bottom disk using strands. These strands can be made to weave around other strands in any way we prefer, but a braid cannot entangle itself. Each point must connect to exactly one strand.

Braids have special mathematical properties. First and foremost, they form a group. In mathematics, a group is a set of objects under an operation which has four properties: closure, associativity, possession of an identity, and inverses. The set of braids B_n form a group where the operation is the composition of braids. The composition of any two braids in the group gives us another braid contained in the group, which defines the closure property of the group. The identity element of the braid group is a braid where all the strands are connected without any weaving (as shown in Figure 2).

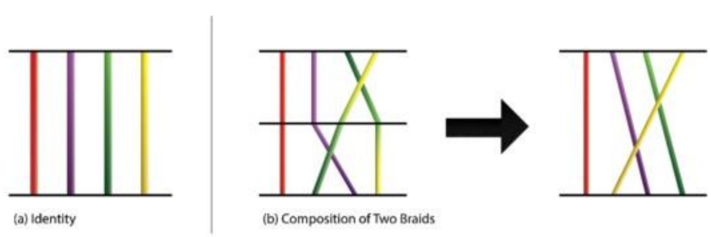


Figure 2

The inverse of a braid B_n is a braid B'_n such that it "undoes" all of the weaving of the initial braid. As a result, the composition of a braid and its inverse gives us the identity braid. Finally, the braid composition is associative because if we have three braids it doesn't matter to compose first one with composition of the last two or first compose the first two and then compose the result with the last one. Thus, we can conclude that braids form a group, since they have all of the properties of a group.

The concept of the Braid Group was generalized so that it could be applied elsewhere. The Braid Group gives us a framework with which we can study the most fundamental building block of life – one's DNA.

DNA is short for deoxyribonucleic acid. It is a complex structure which is found in the cells of all living things. DNA is made of two strands which are held together by chemical bonds. As a result of these bonds, DNA has a double helix structure. If you were to untangle and compose all of the strands of DNA found in the average human body, it would stretch from the Earth to the sun and back almost 600 times!

Since the discovery of DNA in the 1960s, DNA has been an evolving topic of study. DNA is not visible without an electron microscope, and as such abstraction and mathematical modelling have played a significant role in research of DNA. Seeing that the study of DNA is time consuming, expensive, and difficult using an electron microscope, scientists study mathematical models such as the braid group B_2 to understand and classify the complex structure of DNA. B_2 is the exact representation of the linear DNA structure as shown below. As a result, linear DNA has all of the properties of the group B_2.

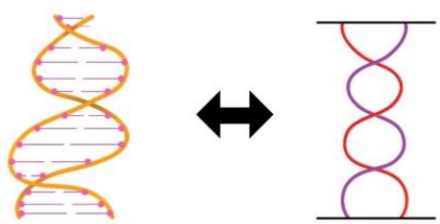


Figure 3. The Transformation of the DNA Structure into a 2-Dimensional Braid Group

Let's discuss the properties of B_2 more closely. Clearly, the braid group B_2 is a braid with two strands. One special property of B_2 is that it has a one-to-one correspondence with the set of integers. More specifically, every braid in B_2 can be represented simply as an integer. Let the identity braid be associated with 0, since it has no weaving. Given a braid, if the left strand goes underneath the right strand, we call this "underlap" and assign number 1 to it. If the left strand goes above the other strand, we call this "overlap" and assign -1 to it. If the strand weaves around the other strand twice, we assign number 2 to it, as shown below.

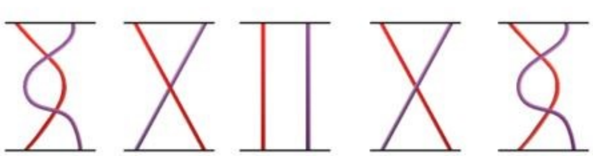


Figure 4. The Representation of Braids in B_2 as Integers.

Hence, we assign an integer to the braid depending on its total amount of weaving. This shows that there is a one-to-one correspondence between the braid group B_2 and the set of integers.

Moreover, we all know that the order in which you add two integers together doesn't make a difference. For instance, 2 + 3 = 5 is exactly the same as 3 + 2 = 5. Similarly, the order in which you add two braids in B_2 doesn't matter. Groups which meet this requirement are knowns as "abelian" or "commutative", hence B_2 is abelian.

Therefore, DNA can also be represented using integers and it must be commutative. This helps identify and classify different types of DNA. In DNA, the manner in which the two strands weave is important because it defines key properties in the human body. And by using the Braid Group, we have a way to represent different weaving patterns using simple numbers. Since DNA is such a complex structure, knowing that it can be represented using simple mathematical models such as the Braid Group and the set of integers, helps scientists understand DNA better.

Biology is not the science that we usually associate with math but the work of early mathematicians was all about looking at the observable world and modelling it using .helps us gain a better understanding of the relation between braids and the most fundamental building block of life, DNA. Using the correlation between DNA and Braid Groups, one can easily verify the importance of mathematics in biology and vice versa. There are many biological systems which are represented using theories of mathematics, and many theories of mathematics are based on the observations of biological systems. The unprecedented progress of research in biological sciences will surelt inspire new mathematical research.

References

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2. Frenkel, Edward. *Love and Math: the Heart of Hidden Reality*. Basic Books, a Member of the Perseus Books Group, 2014.

3. Rettner, Rachael. "DNA: Definition, Structure & Discovery." LiveScience, Purch, 7 Dec. 2017, www.livescience.com/37247-dna.html.

STUDENT TESTIMONIALS

Mslc 2018

The Math and Statistics Centre is a place you definitely don't want to overlook! At first I was really nervous and intimidated by the idea of having to meet a mentor for help. However, the mentors were super welcoming and easy to talk to. The mentors felt like any other friends from class, making it much easier to approach with questions about course content.

Every week I met with a TA who was very understanding and patient and was always willing to adjust her teaching methods to best suit our learning styles. Complex formulas and concepts in class honestly felt like kindergarten work when I was taught by this amazing TA. It was so comforting to know there were people who genuinely cared and took the time to make sure I understood the material thoroughly.

I highly recommend checking the Math and Stats Centre out. It's a comfortable learning place, and a great stop to make if you have questions about content or just simply want to work on some problems with others!

- Gerianne Lozada

The Math and Statistics Centre is definitely worth going to for help. At first I didn’t think much of it, but getting one-on-one help with one of the teaching assistants changed my mind. I was pretty hopeless in stats last semester because I didn’t really understand a thing. During her hours and in seminars, the TA simplified complicated concepts and even changed her teaching strategies based on how we each learned best. She did her best to make sure we left with our questions answered. Going to the Math and Stats Centre really gave me the help I needed to raise my average significantly!

- Arliz Lumbo

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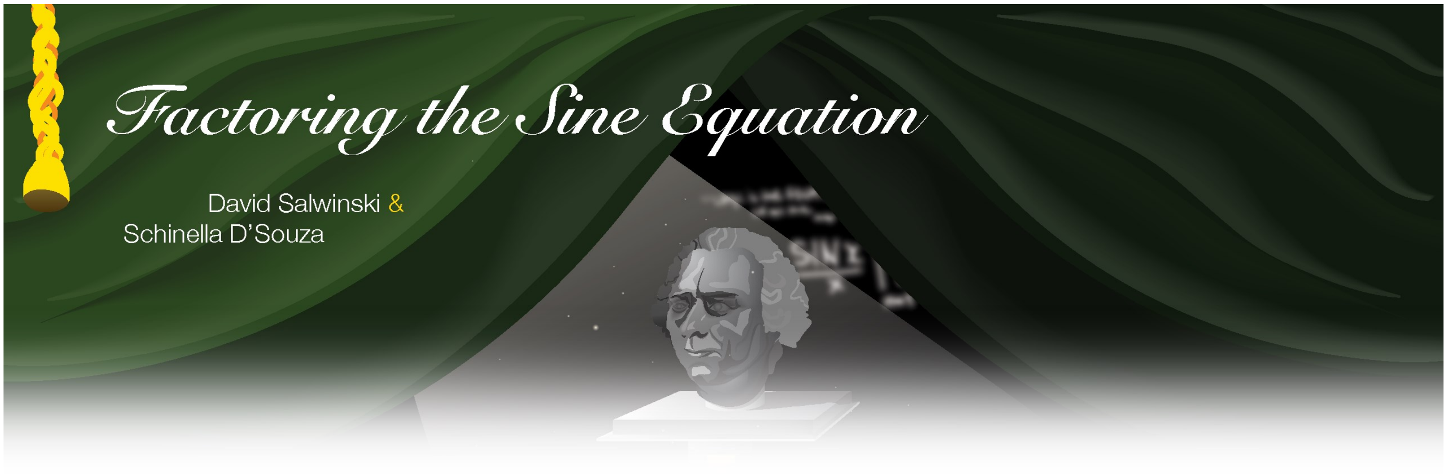
It has helped me to get higher grades by explaining in more depth topics and aiding me to better understand the topic!

- Anonymous



Factoring the Sine Equation

David Salwinski &
Schinella D'Souza

Have you ever wondered if it's possible to factor *sin x* over its roots as you would a polynomial? In 1734 Leonhard Euler, one of history's greatest mathematicians, proposed the following: knowing that a polynomial *p(x)* satisfying p(0)=1 and having nonzero roots at, say, a, b, c, and d, can be factored in the form

$$p(x) = \left(1-\frac{x}{a}\right)\left(1-\frac{x}{b}\right)\left(1-\frac{x}{c}\right)\left(1-\frac{x}{d}\right)$$

It should be possible, given that the function f(x)=sinx/x satisfies f(0)=1 (at least in the limit) and has non-zero roots at ±π, ±2π, ±3π, … , to write

$$\begin{aligned}(x) &= \left(1-\frac{x}{\pi}\right)\left(1-\frac{x}{-\pi}\right)\left(1-\frac{x}{2\pi}\right)\left(1-\frac{x}{-2\pi}\right)\left(1-\frac{x}{3\pi}\right)\left(1-\frac{x}{-3\pi}\right)..\\ &= \left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{2^2\pi^2}\right)\left(1-\frac{x^2}{3^2\pi^2}\right)...\end{aligned}$$

Thus giving the factorization

$$\sin x = x\left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{2^2\pi^2}\right)\left(1-\frac{x^2}{3^2\pi^2}\right)..$$

Incredibly, this equation, known as "Euler's sine product formula", holds for all real x*(!)* and it led Euler to make many amazing discoveries and solve open problems that had stumped his contemporaries. The most famous of these is the Basel problem. Posed in 1644 by Pietro Mengoli and attempted by Leibniz, the Bernoullis, and other leading mathematicians of the day, the Basel problem asked: what is the sum of the reciprocals of all of the perfect squares? Euler stunned the mathematical community when he announced that the answer to this question falls right out of his sine product formula. Expanding the product gives the power series

$$\sin x = x - \left(\frac{1}{\pi^2}+\frac{1}{2^2\pi^2}+\frac{1}{3^2\pi^2}+\frac{1}{4^2\pi^2}+\cdots\right)x^3 + (\ldots)x^5 - \cdots$$

which, when compared coefficient wise to the Taylor expansion

$$\sin x = x - \frac{x^3}{3!}+\frac{x^5}{5!}-\cdots,$$

leads to

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots=\frac{\pi^2}{6}$$

a most surprising result! By equating the coefficients of x^5 and doing a bit of algebra, Euler also discovered that

$$\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots=\frac{\pi^4}{90}$$

Further effort lead him to derive a general formula for the sum of the reciprocals of every even power; see [**2**, pp. 101-133] for more on these sums.

Euler also used his sine product formula to re-derive known results with ease. Letting x=π/2 in the product, for instance, gives

$$\sin\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{6^2}\right)\cdots$$

which, when rearranged, takes the form

$$\frac{\pi}{2} = \frac{2\cdot 2}{1\cdot 3}\cdot\frac{4\cdot 4}{3\cdot 5}\cdot\frac{6\cdot 6}{5\cdot 7}\cdot\frac{8\cdot 8}{7\cdot 9}\cdots$$

This identity is called "Wallis's product for π" after John Wallis, who first obtained it in 1655 with much greater effort [**4**, p. 396] (another derivation has become a standard exercise in first-year calculus textbooks; see [**5**, p. 497] for example). These discoveries, along with others stemming from the sine formula, were Euler's first major contributions to mathematics, and brought him immediate fame at age 28. For more on the subject, along with a detailed account of Euler's intuition, see [**1**, pp. 39-60].

The moral of this story is that sometimes a seemingly out-there idea (such as factoring the sine function), followed by some iffy steps (like applying a formula that works for polynomials to a non-polynomial), may lead to a correct result, and possibly even to amazing new discoveries. In time, the arguments would need to be made rigorous of course, but that might have to wait; for the moment, just obtaining known answers, or ones that are numerically verifiable, is a huge step forward. Euler never managed to provide a rigorous proof of his sine product formula. Indeed, it took about a hundred years for this to be done. When it finally was done by Karl Weierstrass, the long-awaited proof was too advanced to have been accessible to Euler anyway, as it used theorems from a whole new branch of mathematics called complex analysis. Other proofs have been produced since then, but they would most likely all be out of reach to a mathematician of Euler's day. A truly elementary proof of the sine product formula, well within the reach of Euler and any undergraduate, was only published a couple of months ago in the March 2018 issue of "The College Mathematics Journal" by one of the authors of this article; it is accessible online at: https://doi.org/10.1080/07468342.2018.1419703.

Surprisingly, the proof itself is of a similar nature to the standard calculus proof of Wallis's product mentioned above. The moral of this latter story is that sometimes a simple proof to a presumably difficult theorem--even a famous one from nearly 300 years ago that's surely been looked at and thought about countless times by professional mathematicians--could be hiding in plain sight, and it's possible for anyone, even an undergraduate, to spot it and make their own contribution to mathematics. That being said, our advice to every UTSC undergraduate who is serious about studying math is this: take the time to experiment with the formulas and techniques you have learned about in class, attempt your own proofs of the theorems you've been taught, pose your own problems and try to solve them. Don't be shy about breaking the occasional rule or attempting something far-fetched, because you never know when you might stumble upon something significant. It may seem that, at the undergraduate level, everything that could be done already has been--but that's not true! To date there are at least 14 different known proofs of (2) (a paper containing them all may be found at: https://www.uam.es/personal_pdi/ciencias/cillerue/Curso/zeta2.pdf) and even more of (1), with most of them having been discovered in the last 20 years. Mathematics really is a rapidly expanding subject: according to MathSciNet, the largest online database of mathematical research papers, over 57, 000 papers have been published in academic journals in 2017 alone! Think you have a new idea? Search to see if anyone else has already had it at https://mathscinet.ams.org/mathscinet/, and if not, hit up http://math.mit.edu/~cohn/Thoughts/advice.html for advice for amateurs looking to publish.

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