How to Study for Science Courses

Science is the systematic pursuit and application of knowledge, often requiring many different technical skills. As a science student, you may wish to review these skills here.

Computational Thinking and Problem Solving

How to Solve a Math Problem

1. Understand the problem
   a. What is the overall picture?
   b. How familiar are you with this type of problem?
   c. What are you being asked to find or do?

   Here are some tips for understanding math problems:
   • Visualize your understanding by drawing a diagram
   • Exemplify your understanding by recalling similar problems from the past
   • Test your understanding by describing the problem in your own words

2. Understand the details
   a. Of the information you already have, what could be relevant to solving the problem?
   b. Is there information you need but do not have? How could you find it?
   c. Do you understand the notation, vocabulary, and ideas referenced in the problem?

3. Summarize what you know

4. Solve the problem

   Here are some tips for solving math problems:
   • Break the problem down into steps and solve each step
   • Work backwards – before you can get the final answer, what information will you need first?
   • Think about how you solved similar problems in the past
   • Make the problem simpler to increase your understanding of it, e.g. make the numbers smaller
   • Solve multiple different simple versions of the problem to reveal patterns in the results
   • Draw a diagram

Special Case: Proofs

To prove something in mathematics is to show that a particular conclusion follows from a stated premise. To get started, think about the concepts and mathematical definitions that are related to the premises and conclusions.
EXAMPLE: Proving that a definition is satisfied

Q: Prove that the set (2,5) is open.

A: Rewrite the question in terms of the definition of “open.”

Let $x \in (2,5)$ be arbitrary.

Let $d$ be the minimum of $5-x$ and $x-2$.

Then $(x - d, + d) \subseteq (2,5)$.

Therefore, for every $x \in (2,5)$ there exists $d > 0$ such that $(x - d, x + d) \subseteq (2,5)$.

Therefore, $(2,5)$ is an open set.

This was a direct proof – we started by assuming that the set $(2,5)$ was open and then used logical manipulations and deductions to get to the conclusion.

EXAMPLE: Proving general theorems using definitions

Q: Prove that if $f$ and $g$ are both differentiable then $\frac{d}{dx} (fg) = (x) \frac{dg}{dx} + g(x) \frac{df}{dx}$.

A: Write down what the theorem’s premises and conclusions mean in terms of the relevant definitions and then figure out how to get from one to the other.

Suppose that $f$ and $g$ are differentiable. Then

$$\frac{d}{dx} (fg) = \lim_{h \to 0} \frac{(fg)(x+h) - (fg)(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x)}{h} + \lim_{h \to 0} \frac{f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} f(x+h) \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h \to 0} g(x) \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= f(x) \frac{dg}{dx} + g(x) \frac{df}{dx}, \text{ as required}$$

$$= \lim_{h \to 0}$$
This was a direct proof that used the technique of adding and subtracting the same thing to partition the expression into smaller parts that are easier to work with.

**EXAMPLE: Proving a general statement**

\[ x + 1 \text{ if } x < 0 \]

Q: Prove that \( \mathbb{R} \rightarrow \mathbb{R} \) given by \( (x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases} \) is increasing.

A: Rewrite the question in terms of the definition of "increasing."

\[ x + 1 \text{ if } x < 0 \]

Prove that the value of \( \mathbb{R} \rightarrow \mathbb{R} \) given by \( (x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases} \) algebraically increases as \( x \) algebraically increases.

Case 1:
Suppose that \( x_1 < x_2 < 0 \).
Then \( f(x_1) = x_1 + 1 < x_2 + 1 = f(x_2) \).

Case 2:
Suppose that \( x_1 < 0 \leq x_2 \leq 1 \).
Then \( f(x_1) = x_1 + 1 < 0 + 1 = 1 = f(x_2) \).

Case 3:
Suppose that \( x_1 < 0 \) and \( x_2 > 1 \).
Then \( f(x_1) = x_1 + 1 < 0 + 1 = 1 < x_2 = f(x_2) \).

This was a form of direct proof called a proof by cases – we split the question into segments and gave a direct proof for each one.

**EXAMPLE: Proving a general statement**

Q: Prove that \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \).  A: Rewrite the question in terms of the definition of \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \).
Prove that \(1^2 = \frac{1(1+1)(2+1)}{6};\) and that \(1^2 + 2^2 = \frac{2(2+1)(4+1)}{6};\) and that \(1^2 + 2^2 + 3^2 = \frac{3(3+1)(6+1)}{6};\) and that...etc.

Let \(P(n)\) be the statement that \(\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.

Note that \(1^2 = 1 = \frac{1(1+1)(2+1)}{6}\) so \(P(1)\) is true.

Now let \(k \in \mathbb{N}\) be arbitrary and assume that \(P(k)\) is true, i.e. that \(\sum_{i=1}^k i^2 = \frac{k(k+1)(2k+1)}{6}\).

Then \(\sum_{i=k+1}^{k+1} i^2 = (\sum_{i=1}^k i^2) + (k + 1)^2\)

\[= \frac{(k+1)(2k+1)}{6} + (k + 1)^2\]

by the assumption

\[= \frac{(k+1)(2k+1)+6(k+1)^2}{6}\]

\[= \frac{(k+1)(k(2k+1)+6(k+1))}{6}\]

\[= \frac{(k+1)(2k^2+7k+6)}{6}\]

\[= \frac{(k+1)(k+2)(2k+3)}{6}\]

\[= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}\]

So for every \(k \in \mathbb{N}\), \(P(k) \rightarrow P(k+1)\).

Hence, by mathematical induction, \(P(n)\) is true for every \(n \in \mathbb{N}\).

This was a proof by induction – we first proved that \(P(1)\) is true and then showed that if \(P(k)\) is true, \(P(k+1)\) must also be true.

### EXAMPLE: Proving a general statement

Q: Prove that the additive identity for the set \(x \in \mathbb{Z}\) is unique.

A: **Rewrite the question in terms of the definition of “additive identity.”** Prove that \(x \in \mathbb{Z}\) is unique where for every \(z \in \mathbb{Z}\), \(z + x = x + z = z\).

Suppose that there are two integers \(0\) and \(0'\) that are both additive identities for the integers.

Then, by definition,

1. for every \(x \in \mathbb{Z}\), \(x + 0 = x\), and;
2. for every \(x \in \mathbb{Z}\), \(0' + x = x\). In particular,
0' = 0'+0 using (1) = 0 using (2).

So 0' = 0, so the additive identity 0 is unique.

This was a uniqueness proof – we showed that there is only one object with a certain property by assuming that there were two objects and then proving that those two objects were the same.

EXAMPLE: Proving a general statement.

Q: Prove that if \( x \in \mathbb{Q} \) and \( y \not\in \mathbb{Q} \) then \( x + y \not\in \mathbb{Q} \).

A: Rewrite the question in terms of the definitions of \( x \in \mathbb{Q}, y \not\in \mathbb{Q}, \) and \( x + y \not\in \mathbb{Q} \). Prove that if \( \exists p, q \in \mathbb{Z} \) (with \( q \neq 0 \)) such that \( x = \frac{p}{q} \) and \( \exists p, q \in \mathbb{Z} \) (with \( q \neq 0 \)) such that \( y \neq \frac{r}{s} \) then

\[
\exists r, s \in \mathbb{Z} \text{ (with } s \neq 0) \text{ such that } x + y \neq \frac{r}{s}.
\]

Let \( x \in \mathbb{Q} \) so \( \exists p, q \in \mathbb{Z} \) (with \( q \neq 0 \)) such that \( x = \frac{p}{q} \).

Let \( y \in \mathbb{Q} \).

Suppose for contradiction that \( x + y \in \mathbb{Q} \). This means that \( \exists r, s \in \mathbb{Z} \) (with \( s \neq 0 \)) such that \( x + y = \frac{r}{s} \).

But then \( y = \frac{r}{s} - x = \frac{r - p}{q} = (\frac{r}{s} - \frac{p}{q}) \).

Now \( rq - ps \in \mathbb{Z} \) and \( sq \in \mathbb{Z} \) because \( p, q, r, s \in \mathbb{Z} \).

Also \( sq \neq 0 \) because \( q \neq 0 \) and \( s \neq 0 \).

So \( y \in \mathbb{Q} \).

But this contradicts the theorem premise.

So it must be the case that \( x + y \not\in \mathbb{Q} \).

This was a proof by contradiction – we temporarily assumed that the theorem’s conclusion was wrong. Then, we showed that that assumption leads to a contradiction.

Now, back to the steps to solving a math problem.
5. Check your solution
Here are some tips for checking your solution:
• Check the answers in your textbook
• Estimate what the answer could be before solving the problem and then check your solution against the estimate
• Check that your answer is reasonable or in the correct form (e.g. correct units)
• Solve the problem again using a different technique to see if you get the same result

Got the wrong answer? Try solving the problem again or working backwards from the known solution.

6. Reflect on your solution
   a. Why did your procedure work?
   b. How could you change the problem so that your procedure would still work? Would not work?
   c. How could you extend the problem? How could you solve the extension?
   d. Could your procedure be used to solve other problems?
   e. If the problem were a bit different, how could you change your procedure to solve it?
   f. If you had to solve the problem again, would you do anything differently?

Ways to Practise Math
• Discuss math with other people
• Consider how math applies to your daily life
• Complete practice questions

How to Learn Computer Science
1. Understand the basic concepts of the tool you are learning
   Here are some resources to check out:
   • “Crash Course Computer Science Preview” by CrashCourse
   • Cheat-Sheets.org
   • “5 Basic Concepts of Programming” by Double D
   • Dictionary of Algorithms and Data Structures

2. Understand examples of how the tool can be used
   Here are some resources to check out:
   • Algorithms in the "Real World"
   • The Stony Brook Algorithm Repository
   • Here are some tips to try:
     • Review the examples given in your lectures or textbook
     • Review solutions to assignments

3. Solve problems to practise using the tool
   Here are some resources to check out:
   • JFLAP
   • CodingBat
   • 15 Sites for Programming Exercises

4. Understand what the code will do when you run it
   Here are some resources to check out:
   • “Learn Programming in 10 Minutes - 4 Concepts To Read all Code” by Quick Tips
5. Understand what would happen if you changed the code
The key here is to understand the function of each part of your original code and how that part of the code affects other parts of the code.
6. Know how to fix bugs in the code
Here are some resources to check out:
   • Stack Overflow
   • “5 Debugging Tips Every Developer Should Know | Build a Startup #7” by CS Dojo

Collecting, Analyzing, and Interpreting Data

How to Study Statistics
Here are some resources to check out:
   • “Teach me STATISTICS in half an hour!” by zedstatistics
   • UTSC Math & Statistics Support
   • UTSC Stats Resources
   • Statistics and Analysis Calculators - Good Calculators
   • “Statistics with Professor B: How to Study Statistics” by Michelle Benson

Learning to Use Statistical Software
Here are some resources to check out:
   • University of Toronto Libraries Workshops
   • University of Toronto Libraries - Map and Data Library - Software Tutorials
   • University of Toronto Libraries - Map and Data Library - Workshops

Conducting Research
Here are some resources to check out:
   • University of Toronto Libraries - Research
   • University of Toronto Libraries - Research Skills ebooks
   • “The scientific method” by Khan Academy
   • “How to Develop a Good Research Topic” by KStateLibraries
   • “6 Steps to Formulate a STRONG Hypothesis | Scribbr” by Scribbr
   • “How to Create a Strong Research Design: 2-minute Summary | Scribbr” by Scribbr
   • “Analysing, interpreting and presenting data” by Academic Skills, The University of Melbourne

Reading Diagrams and Tables

Pictorial Diagrams
   • What is the diagram’s goal?
   • How does the diagram communicate information?
• Does the diagram contain a size scale?
• Does the diagram have labels?
• Does the diagram have a title, legend, and/or key? If so, what do they tell you about the diagram?
• Does the diagram simplify a complex concept?
• Does the diagram make assumptions about what the viewer already knows?
• Is there text accompanying the diagram?
• Which parts of the diagram are important to remember/understand?
• How successful is the diagram at conveying information?

Relationship Diagrams

• What is the diagram’s goal?
• How does the diagram communicate information?
• Does the diagram have a title, legend, and/or key? If so, what do they tell you about the diagram?
• Does the diagram make assumptions about what the viewer already knows?
• Is there text accompanying the diagram?
• Which parts of the diagram are important to remember/understand?
• How successful is the diagram at conveying information?

Graphs

• What is the graph’s goal?
• How does the graph communicate information?
• Is there any small print accompanying the graph?
• What is the source of the data in the graph?
• Do the data in the graph show any patterns?
• What are the main headings of the graph?
• What are the features of the graph? What does each feature represent?
• Does the graph have a scale? If so, at which values does the scale start and end? In which intervals does the scale increase?
• Does the graph make assumptions about what the viewer already knows?
• Which parts of the graph are important to remember/understand?
• What conclusions can you draw about the data in the graph?
• How successful is the graph at conveying information?

Tables

• What is the table’s goal?
• How does the table communicate information?
• What is the source of the data in the table?
• What do the columns of the table represent?
• What do the rows of the table represent?
• Does the table make assumptions about what the viewer already knows?
• Which parts of the table are important to remember/understand?
• How successful is the table at conveying information?
Reading Math Textbooks

1. Identify which type of information you are reading about (e.g. axiom, theorem)
   a. **Axiom**: a standardized assumption
   b. **Definition**: the meaning of a mathematical word
   c. **Theorem**: a statement about the relationship between certain objects and properties
   d. **Proof**: evidence that a given mathematical statement is true
   e. **Example**: a problem used to demonstrate a mathematical rule

**Note**: You could also be reading about something more abstract than the type of information listed above.

2. Define unfamiliar words and symbols
   As you move through your coursework, it's a good idea to keep a running list of definitions, notations, symbols, and theorems so you have your own personal glossary to refer to. Here is a resource to check out: Mathematics Dictionary & Glossary for Students

3. When you encounter a new concept or idea, consider…
   a. how it relates to your prior knowledge
   b. how it may be similar to concepts or ideas that you have already seen or used
   c. how it may appear in everyday life or practical examples
      i. Here is a resource to check out: Uses of the Mathematics: Top 25 Uses in our Daily Life
   d. where you could use it
   e. where you could not use it
   f. what would happen if you changed its basic assumptions or values

4. When you encounter a practice solution, consider…
   a. how the relevant definitions and properties apply to the problem/solution
   b. how and why each step of the solution is performed
   c. without looking, what the next step of the solution might be

5. Know what you need to know
   A successful math student is generally able to…
   • perform routine calculations
   • adapt proofs and procedures to solve different but related problems
   • show that a definition is satisfied
   • use memorized diagrams or key ideas of definitions, proofs, and theorems to reconstruct the details of a concept or idea
   • understand concepts and ideas both in theory and in relation to examples
   • memorize the main steps of a procedure but reconstruct the details to suit particular problems or situations

Taking Notes in Math Class

1. Divide your notepaper into 3 columns, where the second column is wider than the first and third columns
2. At the top of the first column, write the header “Key Words/Rules”
3. At the top of the second column, write the header “Examples/Problems”
4. At the top of the third column, write the header “Explanations/Descriptions”
5. When your professor mentions a key word, idea, or formula, write it down in the first column.
6. When the professor uses an example to illustrate that key word, idea, or formula, write the example down in the second column.
7. In the third column, write down the explanation accompanying the example (e.g. why is each step performed, etc.)

Here are some tips to try:

- Write large numbers using commas instead of spaces (e.g. 45,000 instead of 45 000) to remind yourself that the groups of digits are part of the same number.
- Use semicolons to separate numbers in a list (e.g. 65; 64.5; 64.51)
- When handwriting notes, underline letters and symbols instead of italicizing them.
- Copy letters and symbols down exactly as the professor writes/draws them.

Making Diagrams and Tables

1. Remember the golden rule of diagram-making: People should be able to understand your diagram even if they have not read the text or explanation accompanying it.
2. Elements of a good diagram:
   a. simple
   b. includes a title
   c. clearly labelled
   d. referenced in accompanying text (if applicable)
   e. cites sources of data (if applicable)
   f. unconventional aspects of the diagram (e.g. unconventional format) are acknowledged and justified by the creator.
3. For pictorial diagrams:
   a. You do not have to create a perfect rendition of the object or phenomenon you are modelling! Only include details that are relevant to what you want your diagram to communicate.
   b. Here is a resource to try: Canva
4. For relationship diagrams:
   a. Here is a resource to try: diagrams.net
5. For graphs:
   a. Which type of graph should you use? That depends on the type of data you want to model! For help, check out the Data Visualization Guide by University of Toronto Libraries.
   b. Here is a resource to try: Graph maker tool
6. For tables:
   a. Before making your table, consider how many columns and rows you will need to display your data.
   b. Make sure your columns and rows have clear headings.
   c. Here is a resource to try: Microsoft Excel.

Studying Diagrams

How to Memorize a Diagram

1. Photocopy the diagram and remove the labels.
2. Close your eyes and place your finger somewhere on the diagram
3. Open your eyes
4. Label whichever part of the diagram your finger is touching (or is closest to)

**Alternatively:**

1. Write the diagram labels down on a series of flashcards
2. Photocopy the diagram and remove the labels
3. Pick a random flashcard
4. Identify which part of the diagram aligns with the label on the flashcard

**How to Memorize a Process**

1. On one side of a flashcard, write the name of a step in the cycle
2. On the other side of the flashcard, write the number of the step (e.g. Step 1)
3. Make a flashcard for each step
4. Draw a random flashcard from your pile
5. If you see the name of a step, recite the number; if you see the number of a step, recite the name
6. Keep going until you have memorized all the flashcards

Next,

1. Cover your notes
2. Explain the entire process out loud to yourself

Finally,

1. Cover your notes
2. Draw a random flashcard from your pile
3. Explain the step written on the flashcard

**Lab Skills**

**Before a Lab Session**

1. Understand the lab procedure
   a. What is the end goal of the lab? What information or substance are you ultimately seeking?
   b. What steps will you take to reach that end goal?
   c. When will you perform each step? What is their order?
   d. Where will you perform each step? Will you need to be in a certain area of the lab?
   e. How will you perform each step?
   f. Who will be in the lab with you? Will you work with a partner? Will you need to consult your TA/supervisor during the lab?

Here are some resources to try:

- Virtual Lab Simulations
- JoVE (for lab procedure videos)
- Open Educational Resources
• UTSC DPES (for lab procedure videos)

2. Understand the safety and equipment issues relevant to the lab procedure
   a. Here is a resource to try: “Lab Techniques & Safety: Crash Course Chemistry #21” by CrashCourse

3. Understand the theoretical concepts related to the lab procedure
   a. How is the lab content of your course related to the lecture content?
   b. Think about each step of the lab procedure. Why is that step part of the procedure?
      What would happen if you did not perform it?
   c. Think about each step of the lab procedure. Why must you perform that step a certain way? What would happen if you performed it a different way?
   d. What is the theory or theoretical knowledge that informs the lab procedure?

4. Complete any assigned preparatory work Here are some resources to try:
   • “Chemistry Lab Skills: Maintaining a Lab Notebook” by UTSC DPES
   • Biology Resources
   • Chemistry Resources
   • Environmental Science Resources
   • Neuroscience Resources
   • Health Studies Resources
   • Math Resources
   • Psychology Resources
   • Physics Resources

5. Make a time management plan
   a. How much time will each step of the lab process truly need?
   b. Can you change the order of the steps to make the process more efficient?
   c. Can you combine certain steps or perform them together?

   Here is a tip to try:
   • Write down exactly what you plan to do within the first 20 minutes of the lab and why you will do those things. Doing so can help you identify which parts of the lab procedure are important and will require a lot of time and which parts are minor steps that can be finished quickly.

During a Lab Session

Here are some resources to check out:

• “3 Ways to Immediately Improve Your Attention to Detail” by Attention to Detail
• “How to Take Faster Notes - College Info Geek” by Thomas Frank

CAUTION!
Shorthand and abbreviated notetaking systems can help you take notes during a fast-paced lab session, but if your TA/prof will grade your notebook, make sure to clean your writing up afterwards to make it legible.

After a Lab Session

Here are some resources to check out:

• “How to Write a Lab Report” by Apologia
Need Some More Help?

Anthropology

- Howtostudy.org - Anthropology
- Valuable Online Resources for Students of Anthropology

Astronomy

- Howtostudy.org - Astronomy
- Sky & Telescope

Biology

- Howtostudy.org - Biology
- Biology Resources for Students

Chemistry

- Howtostudy.org - Chemistry
- Master Organic Chemistry

Computer Science

- Howtostudy.org - Computer Science
- A list of open-access resources to learn computer science

Environmental Science

- Howtostudy.org - Geology
- Environmental Science - Virtual Lab and Science Resource Directory

Health Studies

- Open Educational Resources (OERs): Health Sciences
- Resources for medical students

International Development Studies

- Howtostudy.org - Geography
- Development studies resources for students

Mathematics

- Howtostudy.org - Math
- Canadian Mathematical Society - Resources for students
Neuroscience

- Neuroguide.com - Best Bets
- UBC Neuroanatomy

Paramedicine

- Paramedic Tutor
- Paramedicine - Research guide

Physics

- Howtostudy.org - Physics
- Physics and Astronomy Websites

Psychology

- Howtostudy.org - Psychology
- Psychology Student Resources

Statistics

- Statistics Help Resources
- Stat Trek

For more support, come visit us at the Academic Advising & Career Centre!
References

This tip sheet was adapted from the following sources:


www.utsc.utoronto.ca/aacc  Last updated: January 2022