Quantum Eraser

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It was in 1935 that Albert Einstein, with his collaborators Boris Podolsky and Nathan Rosen, exploiting the bizarre property of quantum entanglement (not yet known under that name which was coined in Schrödinger (1935)), noted that QM demands that systems maintain a variety of 'correlational properties' amongst their parts no matter how far the parts might be separated from each other (see 1935, the source of what has become known as the EPR paradox). In itself there appears to be nothing strange in this; such correlational properties are common in classical physics no less than in ordinary experience. Consider two qualitatively identical billiard balls approaching each other with equal but opposite velocities. The total momentum is zero. After they collide and rebound, measurement of the velocity of one ball will naturally reveal the velocity of the other.

But the EPR argument coupled this observation with the orthodox Copenhagen interpretation of QM, which states that until a measurement of a particular property is made on a system, that system cannot, in general, be said to possess any definite value of that property. It is easy to see that if distant correlations are preserved through measurement processes that 'bring into being' the measured values there is a prima facie conflict between the Copenhagen interpretation and the relativistic stricture that no information can be transmitted faster than the speed of light. Suppose, for example, we have a system with some property which is anti-correlated (in real cases, this property could be spin). If we measure one part of the system, and find that the correlated property of that part is say, $+1$, then we know that if we were to measure the distant part of the system it would reveal that property to have value -1 (to preserve the negative correlation). It is rather as if, in our billiard ball example, we knew that the total momentum was zero but that we believed that neither ball had a particular velocity until we measured it.

Then there is an evident problem of how the other ball 'knows' what value we got for its partner's velocity. If the system's components are separated to a sufficient distance there can be no possibility of any ordinary sort of 'communication' between the parts (i.e. any communication process which operates at light speed or below).

The obvious answer, championed in Einstein *et al.* (1935) (as well as the classical, commonsense picture of the world), was that there are some hidden elements of reality that both parts of the system carry with them as they are separated; these hidden elements are initially correlated and maintain that correlation until they are measured, at which point they merely reveal a value that they had possessed all along. Einstein, Podolsky and Rosen argued that QM must therefore be incomplete. This natural solution was spectacularly criticized by the work of John Bell who showed that if QM was correct in its predictions then any version of the EPR model of 'carried correlations' must be incorrect (see Bell (1964); see also Bell (1987) which contains both the original paper as well as additional papers bearing on EPR). What Bell showed was that the measurement statistics of a wide range of reasonable hidden variable theories must obey some form of a relatively simple algebraic relation now called the 'Bell inequality'.

More precisely, Bell showed that no *local* carried correlation theory can be correct. If we allow that the parts of the system can 'communicate' instantaneously across any distance then we can maintain a hidden variable theory, since the separated parts could through their intercommunication so to speak manipulate the measurement statistics to bring them in line with those of ordinary QM. And in fact there is such a non-local hidden variable theory, developed by David Bohm (see 1952; see also Bohm (1980), Bohm and Hiley (1993)). In Bohm's theory all the positions and trajectories of the particles in a system are always determinate but there is non-local 'communication' via the so-called quantum potential. This is a new kind of field, mathematically derived from and implicit in the Schrödinger equation, that, in essence, carries information rather than energy and which is able to 'guide' the particles. The nature of this communication is mysterious however and it remains somewhat unclear whether Bohm's theory can be satisfactorily extended to the relativistic context of quantum field theory, although recent work is encouraging. In any case, Bohm's theory certainly does not vindicate the EPR intuition and in fact champions the 'spooky action at a distance' that Einstein deplored.

A number of loopholes are possible to contemplate which could make the world safe for locality. For example, one could imagine that the measurements somehow depend on pre-established experimental protocols. This loophole could be closed by having the protocols chosen randomly and rapidly enough before the measurement that communication between the parts of the system is impossible. A number of other loopholes have been noticed and in a number of cases experiments mounted that try to close them have been attempted. In recent years, from 1972 to 2013, a series of experiments directly aimed at testing the Bell inequality under ever more stringent conditions which eliminate more or less improbable possible loopholes that might save locality have been performed (for a general discussion of the nature of these tests and their significance see Shimony (2013); Wikipedia has a nice compilation). The experiments have all thus far vindicated QM.

QM demands that the distant parts of systems remain 'aware' of what is happening to the other parts. This is an information link but a most peculiar one: no information, in the sense of bit capacity, can be transmitted over the link. The resolution of this 'paradox' requires us to distinguish causal chains from information links. Ordinary information theory reduces information transmission to causal connection, but it seems there is a more fundamental sort of information laden connection in the world. It is possible to view information as the basic element at work here, so that causal processes come to be seen as just one, albeit particularly visible and salient, form of information link. The paradox of correlated systems is resolved if we note that if it were possible to transmit information by manipulation of the distant parts of some QM correlational system one could set up a causal process from one to the other. This is ruled out by relativity theory. But if other sorts of information links are envisaged, then of course an information link can remain in despite of the absence of any causal link.

It is also interesting that since the source and, in some sense, maintenance, of the correlations between the distant parts of some system are dependent on fundamental physical conservation laws, such as the conservation of momentum, the constraints imposed on the world by these laws are not always enforced by causal processes. It has always seemed remarkable to me that laws can constrain the world in 'abstract' ways so that the mechanisms of their observance vary from system to system (each candidate perpetual motion machine can be seen to fail, but the failure in each case depends upon the details of the machine at issue). It is surely significant that the 'mechanisms of law observance' transcend the realm of causal process and seem to enter a more general sphere of pure informational commerce[1].

But the notion of 'pure information' I want to develop can be better illustrated in a less exotic setting, through a simple discussion of the famous two-slit experiment. A beam of photons, electrons, atoms or whatever is directed towards an appropriately separated pair of slits in an otherwise opaque surface. A detector screen is set up behind the slits. QM predicts, and less ideal but more practical experiments amply verify, that the 'hits' on the screen will form an interference pattern, which results in some way from the interaction of the two possible paths an element of the test beam can take to the screen.

More particularly, the QM formalism demands that the atoms, say, in the beam be represented as a superposition of the states associated with each spatial path, for example:

$$\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2) \tag{1}$$

where the coefficient, $1/\sqrt{2}$, is a normalization factor required to insure that the output probabilities upon measurement remain between 0 and 1, the root because probabilities are given by the square of the system's state function, ψ. Here, ψ represents the 'total state' of the particle which passes through the apparatus, ψ_1 represents the particle taking the top slit, call this path 1, and ψ_2 represents the particle taking the bottom slit, call this path 2. The probability that the screen will be hit in a certain region, r, is given by a variety of formal mechanisms.

In general, probabilities are given by what is called the inner product, which can be thought of as the overlap of the two states involved, thus:

$$\langle \psi | P_r \psi \rangle \tag{2}$$

Here, P_r is an operator which transforms a state into a state in the space of states representing the system (a particle for example) being found at position r. If we imagine that the system is already in a state corresponding

[1] There are other instances of laws which do not operate via imposing causal mechanisms on the world, such as Pauli's exclusion principle and, from a certain point of view, laws like the perfect gas law and Newton's law of gravitation. Although we now know that the two latter laws are based on underlying causal mechanisms, they had their legal status prior to our discovering this and could be laws even if we never had discovered their underlying explanation (see van Fraassen (1980), especially ch. 5)

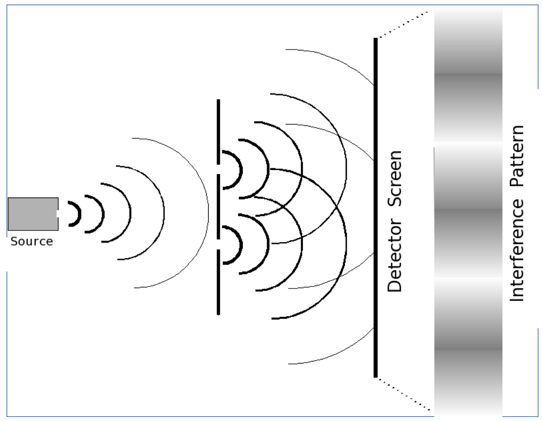

Figure 1: 2 Slit Experiment

to being at position r, the overlap between this state and the projection will be a total or perfect overlap, and the probability will come out to be 1. The details of how this works mathematically don't matter to us. We can simply regard P_r as a 'machine' which transforms states appropriately and take the mathematics as just a device which delivers probabilities. If we write out the inner product in full, we get

$$\left\langle \frac{1}{\sqrt{2}}(\psi_1 + \psi_2) | P_r \left[\frac{1}{\sqrt{2}}(\psi_1 + \psi_2) \right] \right\rangle \tag{3}$$

In QM, operators which correspond to observable quantities such as P_r (position) are linear[2]; so given the properties of the inner product we can expand (3) into:

$$\frac{1}{2} \left[\langle \psi_1 | P_r \psi_1 \rangle + \langle \psi_2 | P_r \psi_2 \rangle + \langle \psi_1 | P_r \psi_2 \rangle + \langle \psi_2 | P_r \psi_1 \rangle \right] \tag{4}$$

The first two terms respectively represent the probability of the particle being in region r if it takes path 1 or if it takes path 2. The final two terms are the unavoidable 'cross terms' which, at least mathematically, account for the interference between the two paths. Schematically, the situation can be pictured as in Figure 1. The darker regions of the interference pattern on the

[2] We also appeal to this feature of the inner product: $\langle c\psi | d\phi \rangle = c^* d \langle \psi | \phi \rangle$ where c^* is the complex conjugate of c. This complication is forced on us because scalars in QM can be either real or imaginary numbers. In this particular case, the coefficient of $1/\sqrt{2}$ is 'pure real' and hence is equal to its own conjugate.

right of the figure correspond to a greater number of detected particles. Right behind the slits we find a very low number of particles, contrary to classical expectations for particles (though perfectly in line with the expectations for wave phenomena).

As everyone knows but which is still amazing, the interference pattern disappears if we have some way of determining through which slit a given atom has passed on its way to the screen. This is sometimes explained in terms of the causal disturbance of the atom's state which such a measurement will involve, and sometimes it is said that such a disturbance is unavoidable and is the proper account of this aspect of the two-slit phenomena (and, in general, of the uncertainty relations in QM).

But there is no need to posit disturbance in order to explain the loss of the interference pattern; mere information about which path the atoms take will suffice. For suppose that there was a perfect detector that could determine which path an atom has taken without altering the atom's state. Such a detector would be capable of only two detection states, let's say T, B (for top and bottom slit respectively), and the output of the detector would be perfectly correlated with the components of the atomic state, ψ_1 and ψ_2. The joint system of atom plus detector state, *after* detection, which I'll label ψ_d, would be written as a superposition of tensor products (again, the details of the tensor product machine don't matter to us, though I provide some of the manipulation rules, which in this case are not particularly complicated, as needed below, as follows:

$$\psi_d = \frac{1}{\sqrt{2}} [(\psi_1 \otimes T) + (\psi_2 \otimes B)] \tag{5}$$

Now if we wish to compute the probability of finding an atom in region r, we require an operator that works on the so-called tensor product space of the atom plus detector. Since we are only interested in measuring the position of the atom and have no wish to do anything at all to the detector, this operator is $P_r \otimes I$, where I is the identity operator (i.e. for any ψ, $I\psi = \psi$)[3]. The basic form of our probability equation is just as above, but taking into account the existence of the detector; the probability of finding the particle in region r is now:

$$\psi_d|(P_r \otimes I)\psi_d \tag{6}$$

[3] And, in general, if O and P are operators we have $(O \otimes P)(\psi \otimes \phi) = O\psi \otimes P\phi$.

Writen out in full this gets rather messy:

$$\left\langle \frac{1}{\sqrt{2}} \left[(\psi_1 \otimes T) + (\psi_2 \otimes B) \right] \middle| (P_r \otimes I) \frac{1}{\sqrt{2}} \left[(\psi_1 \otimes T) + (\psi_2 \otimes B) \right] \right\rangle \tag{7}$$

but if we abbreviate $(\psi_1 \otimes T)$ simply to X, $(\psi_2 \otimes B)$ to Y and the operator $(P_r \otimes I)$ to O, the fundamental form will become easier to discern:

$$\left\langle \frac{1}{\sqrt{2}}(X+Y) | O \left[\frac{1}{\sqrt{2}}(X+Y) \right] \right\rangle \tag{8}$$

This is entirely analogous to (3) above. However, when (8) is expanded the cross terms take on a distinct form. The first step gives us:

$$\frac{1}{2} \left[\langle X|OX \rangle + \langle Y|OY \rangle + \langle X|OY \rangle + \langle Y|OX \rangle \right] \tag{9}$$

The expansion of just the first and last term (which is a cross term) of (9) should be enough to reveal what will happen to the probabilities in this case.

$$\begin{aligned} \langle X|OX \rangle &= \langle (\psi_1 \otimes T) | (P_r \otimes I) \psi_1 \otimes T \rangle \\ &= \langle (\psi_1 \otimes T) | P_r \psi_1 \otimes T \rangle \\ &= \langle \psi_1 | P_r \psi_1 \rangle \times \langle T|T \rangle \end{aligned} \tag{10}$$

This follows from the definition of the inner product in the tensor product space, which is: $\langle (\psi_1 \otimes \phi_1) \,|\, (\psi_2 \otimes \phi_2) \rangle = \langle \psi_1|\psi_2 \rangle \times \langle \phi_1|\phi_2 \rangle$. Since all our state vectors are normalised, $\langle T|T \rangle = 1$ and (10) is simply the probability of the particle being in region r if it took the first path. As we would expect, the detector state has no effect on this probability.

The situation is quite different with the cross terms. Consider

$$\begin{aligned} \langle X|OY \rangle &= \langle (\psi_2 \otimes B) | (P_r \otimes I)(\psi_1 \otimes T) \rangle \\ &= \langle (\psi_2 \otimes B) | (P_r \psi_1 \otimes T) \rangle \\ &= \langle \psi_2 | P_r \psi_1 \rangle \times \langle B|T \rangle \end{aligned} \tag{11}$$

Note that this cross term is accompanied by the factor $\langle B|T \rangle$ (the other cross term of (9) will be accompanied by $\langle T|B \rangle$). But in a perfect detector, distinct indicator states are orthogonal (there is zero overlap between the detector states), which is to say that these inner products have the value 0 and the interference terms thus disappear. The probability that the atom will be found in region r is now just the sum of the probability of its being in r if it takes the first path and the probability of its being in r if it takes the second path.

This is of some interest to those who need to be reminded that complementarity is not the result of the clumsiness of measurement, but is rather an intrinsic and ineradicable feature of QM. The mere fact that our detectors carry the relevant information is sufficient to destroy the interference effects, whether or not the detector in some way 'disturbs' the system under measurement. The kind of information at issue here is not bit capacity but the semantically significant correlation of 'distinct' physical systems, where there is no requirement that the correlation be maintained by some causal process connecting the two systems (which is not to say that there is no influence of one part of the system on another but there is no transfer of energy characteristic of causal processes). The properties of the QM system are of course fully explicated by the structure of the wave function describing the whole system, but what we seek to understand is the connection between the purely mathematical, abstract space in which the wave function evolves and the solidly real, 'spread-out' physical space in which the properties of the system are actually discovered by us.

This remarkable feature of QM is made more apparent by my final excursion into the formalism, only slightly more complex. The notion of a perfect detector suggests the possibility of retrieving the original interference patterns simply by erasing the information within the detector. Since the atomic states have not been altered by the initial operation of the ideal detectors, this would appear to be at least theoretically feasible. To speak figuratively: the atoms, now far along on their way towards the screen upon which their position will eventually be recorded, have no idea whether their paths have been registered or not. Such an interference retrieval device is called a quantum eraser (see Scully and Drühl (1982); Scully *et al.* (1991); for an interesting discussion of the nature and significance of quantum erasers see Davies (1996), ch. 7).

The simplest imaginable or naive quantum eraser would be modelled by some operator that transformed either of the detector states,T or B, to the same neutral third state, say a ground state, G. Call this hypothetical operator, $\mathfrak{R}$, the reset operator. Then we could represent the eraser as $\mathfrak{R}$ acting on the detector states thus: $\mathfrak{R}(T) = \mathfrak{R}(B) = G$. Since $\mathfrak{R}$ acts only on the detector it would be represented in the tensor product space as $(I \otimes \mathfrak{R})$. We could choose to turn on the eraser or not. If we did, its action on ψ_d would evidently be this:

$$(I \otimes \mathfrak{R})\psi_d = \frac{1}{\sqrt{2}}[(\psi_1 \otimes \mathfrak{R}(T)) + (\psi_2 \otimes \mathfrak{R}(B))]$$
$$= \frac{1}{\sqrt{2}}[(\psi_1 \otimes G) + (\psi_2 \otimes G)] \tag{12}$$

Now, upon expansion, the terms $\langle B|T\rangle$ and $\langle T|B\rangle$, which previously eliminated the interference, become $\langle G|G\rangle$, which we may suppose has unit norm and so the interference terms are back!

The quantum eraser just outlined might be called a perfect eraser, and I hope it does illustrate the idea behind the eraser, but unfortunately it is entirely impossible as it is presented here. No such operator as $\mathfrak{R}$ is allowable in QM since it violates the laws governing the time-evolution of quantum mechanical states. The temporal evolution of a quantum system is governed by the system's time-dependent Schrödinger equation, but this evolution can also be given by a set of operators, U_t, such that ψ_t (i.e. the state at time t) is $U_t\psi_0$ (where ψ_0 is the initial state of the system at time zero). These operators are unitary, and this entails (among other things) that they preserve orthogonality (in fact, $\langle X|Y\rangle = \langle U_tX|U_tY\rangle$). Thus, there is no way that our two orthogonal detector states, T and B, could both be reset to the same state, G[4].

We should have had our doubts about this version of the quantum eraser anyway. If there was a perfect quantum eraser it would be a truly magical machine. For consider. We can choose to activate (or not) the eraser at any time before the relevant atom reaches the screen (probably even after it reaches the screen) and, in principle, the screen could be a goodly distance away. But so long as we do activate the eraser the atom will be 'directed' to a region of the screen compatible with interference; if we do not activate the

[4]One might, perhaps, entertain some doubts about this argument since, notoriously, the normal time evolution of a quantum state seems to fail in the case of measurement where the so-called collapse of the wave function occurs. There is no question that two orthogonal states can both 'collapse' to the same state. E.g. the result of a measurement of spin in the z-direction of an electron already prepared to be spin-up in the x-direction could be spin-down in the z-direction; the very same result could, of course, be obtained from a measurement of a spin-down in the x-direction electron. But in the case above, we maintain the superposition of states which is characteristic of the normal time-evolution of quantum states; we did not invoke any collapse of the wave function in the operation of the eraser and, it seems, any such collapse would necessarily eliminate one of the terms of the superposition and thus would also eliminate any possibility of interference.

eraser the atom goes to a non-interference part of the screen. Of course, such regions overlap so those limited to observing the screen might only receive statistical evidence about whether the eraser is on or off but after a time it would become clear, and sometimes the atom would hit the screen in a place that would make it very unlikely that the eraser had been turned on. Now suppose that whether or not the eraser is activated will be determined by a randomizing device which renders its decision just prior to eraser activation or non-activation. To make things vivid, if rather impractical, let's say the randomizer is a certain roulette wheel in Monte Carlo ('red' means turn the eraser on, 'non-red' means leave it off).

Let us also suppose that the distance between screen and slits plus detector plus eraser apparatus and the delay between the atom's passing the slits and activation (or not) of the eraser are considerable enough to ensure that the two events of eraser activation and atom detection on the distant screen are space-like separated. In that case, there is a moving observer for whom the atom will hit the screen before the quantum eraser is activated. Such an observer will then know (at least to a degree better than chance) the outcome of the spin of the roulette wheel before the wheel has been spun. More particularly, there are certain regions of the screen such that if an atom is registered in them then it is very likely that the roulette wheel will come up non-red. But I take it as given that no one, especially someone with no particular knowledge about the roulette wheel in question, can have this sort of knowledge.

Worse still, the perfect quantum eraser would permit superluminal signalling. For suppose that instead of randomly engaging the eraser we try to use it as a kind of Morse code transmitter. Again, because of the overlap between interference and non-interference regions, we might have to send the same signal repeatedly to ensure a reasonable likelihood of proper reception, but that is no difficulty of principle. By waiting until the last second, as it were, to activate (or not) the eraser, we can direct a distant atom to various regions of the screen with some level of control. It is this possibility, perhaps, which appalled the physicist E. T. Jaynes. He wrote about a related quantum device but I adjust the quote here to accord with the eraser example: 'by applying [the eraser] or not ... we can, at will, force [the atoms] into either (1) a state with ... no possibility of interference effects ... (2) a state [in which] interference effects are then not only observable, but predictable. And we can decide which to do after the [atoms have passed the slit/detector] so there can be no thought of any physical influence on the [atoms]' (1980, p.

41; this quote appears, and is discussed, in Scully *et al.* (1991)). Jaynes goes on to insult QM as having 'more the character of medieval necromancy than of science' (p. 42).

Such remarks might seem to presuppose that a perfect quantum eraser is possible, but as we have seen this is in fact not the case. That is to say, we have discovered that what I called the *perfect* eraser is impossible. However, despite the considerations given above, quantum erasers are possible. Their construction is just a little more complicated than appeared at first sight.

Consider again the state of our atoms as they proceed through the quantum eraser apparatus. After passage the state of the system is that given in (5) above. We cannot reset the detector in the way required for the perfect eraser, but it will suffice if we can discover appropriate, possible states that the detector can achieve which will still allow the eraser to function. Such states are possible. What is needed is a mathematical trick, which is the heart of the Scully et al. scheme for eraser construction. Define four new states as follows:

$$\psi_+ \equiv \frac{1}{\sqrt{2}}(\psi_1 + \psi_2) \tag{13}$$

$$\psi_- \equiv \frac{1}{\sqrt{2}}(\psi_1 - \psi_2) \tag{14}$$

$$G_+ \equiv \frac{1}{\sqrt{2}}(T + B) \tag{15}$$

$$G_- \equiv \frac{1}{\sqrt{2}}(T - B) \tag{16}$$

Since any linear combination of quantum states is a quantum state, these are all perfectly legitimate states of our hypothetical system. They are all observable states[5]. The states G_+ and G_- are to be thought of as states the detector can enter through the operation of the eraser. Furthermore, and crucially, the original state, ψ_d, can be written in terms of our new states, as follows:

$$\psi_d = \frac{1}{\sqrt{2}}\left[(\psi_+ \otimes G_+) + (\psi_- \otimes G_-)\right] \tag{17}$$

This can be verified algebraically, from the properties of the tensor product. The crucial properties are that $c\psi \otimes d\phi = cd(\psi \otimes \phi)$ for scalars c, d (in QM

[5] In the quantum terminology, there are Hermitian operators of which they are eigenstates.

scalars can be either real or imaginary numbers) and that $(\psi_1 \otimes \phi_1) + (\psi_2 \otimes \phi_2) = (\psi_1 + \psi_2) \otimes (\phi_1 + \phi_2)$. The proof of (17) then follows by expansion of its right term, thus:

$$\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(\psi_1 + \psi_2) \otimes \frac{1}{\sqrt{2}}(T + B)\right] + \left[\frac{1}{\sqrt{2}}(\psi_1 - \psi_2) \otimes \frac{1}{\sqrt{2}}(T - B)\right]$$

Gathering terms as per the above properties this becomes

$$\frac{1}{\sqrt{2}}\Big[\frac{1}{2}\left[(\psi_1 \otimes T) + (\psi_1 \otimes B) + (\psi_2 \otimes T) + (\psi_2 \otimes B))\right] + \frac{1}{2}\left[(\psi_1 \otimes T) - (\psi_1 \otimes B) - (\psi_2 \otimes T) + (\psi_2 \otimes B)\right]\Big]$$

We notice that several terms here cancel out by subtraction to leave us with

$$\frac{1}{\sqrt{2}}\left[\frac{1}{2}\left[(\psi_1 \otimes T) + (\psi_1 \otimes T) + (\psi_2 \otimes B) + (\psi_2 \otimes B)\right]\right]$$

Now combining the double terms and diving by the factor of one-half yields

$$\frac{1}{\sqrt{2}}\left[(\psi_1 \otimes T) + (\psi_2 \otimes B)\right] = \psi_d.$$

So, as it must, this state exhibits no interference since the cross terms contain the vanishing $\langle G_+|G_-\rangle$ and $\langle G_-|G_+\rangle$.

But suppose we ask, what is the probability of the particle being in region r *given* that the detector is in the state G_+? On the assumption that the detector is in G_+ the second term on the right side of (17) side must vanish and the probability will be calculated from the state $\psi_+ \otimes G_+$. This calculation proceeds normally; so the probability of the particle being in region r given that the detector is in state G_+ is:

$$\langle \psi_+ \otimes G_+|(P_r \otimes I)(\psi_+ \otimes G_+)\rangle \tag{18}$$

which quickly reduces to

$$\langle \psi_+|P_r\psi_+\rangle \times \langle G_+|G_+\rangle \tag{19}$$

It is easy to see that $\langle G_+|G_+\rangle$ is equal to 1, so the probability we seek is simply $\langle \psi_+|P_r\psi_+\rangle$. The expansion of this inner product is however very interesting. Given the definition of ψ_+, this probability expression is just (3) above. That is, we have recovered the original two-slit configuration with its interference effects despite the operation of the detector and we have done so via the operation of the eraser!

What happens to the probability on the assumption that after the operation of the eraser the detector goes into its other possible state G_-? This probability will be equal to $\langle \psi_- | P_r \psi_- \rangle$, which is:

$$\left\langle \frac{1}{\sqrt{2}}(\psi_1 - \psi_2) | P_r \frac{1}{\sqrt{2}}(\psi_1 - \psi_2) \right\rangle \tag{20}$$

and this expands by the following steps:

$$\begin{aligned} &= \frac{1}{2} \langle (\psi_1 - \psi_2) | P_r (\psi_1 - \psi_2) \rangle \\ &= \frac{1}{2} [\langle \psi_1 | P_r \psi_1 \rangle - \langle \psi_2 | P_r \psi_2 \rangle - \langle \psi_1 | P_r \psi_2 \rangle + \langle \psi_2 | P_r \psi_1 \rangle] \end{aligned} \tag{21}$$

Here too we have interference effects, but they are the opposite of those attendant upon (19). The *sum* of these two interference effects produces a pattern at the screen identical to the no-interference pattern produced by the operation of the detector without the eraser.

So, have we produced a quantum eraser with the magical properties discussed above? The best answer seems to be 'yes and no'. We have a quantum eraser all right, but it cannot be used in any of the ways imagined above. This is because the peculiar effects of the eraser are evident only if we know which state the detector is in after the passage of each atom and there is no way to get this information to someone in the vicinity of the screen except by ordinary means, which precludes such things as superluminal signalling or 'predicting' the outcomes of roulette wheels. In order to use the eraser to achieve such ends, the eraser would have to send the detector into a determinate state or our wishing, and this, we have seen, it simply cannot do. On the other hand, from the point of view of the universe, as it were, something quite mysterious is going on. For the atoms are 'responding' to the operation of the eraser and they are doing so instantaneously across (in principle) any distance.

The story is not quite over. It might be objected that the idea of the eraser can't even get off the ground since it presupposes the existence of 'perfect detectors' which are in reality entirely impossible. However, perhaps surprisingly, it is not the impossibility of perfect detectors which do not disturb the state of their target atom which destroys the idea of the quantum eraser. As outlined in Scully *et al.* (1991), it is possible to construct a 'micromaser cavity' that will guarantee that an excited atom will de-excite (via emission of a photon of characteristic wavelength) while passing through the cavity. The emission of the photon will have no significant effect on the 'centre-of-mass' wave function of the atom, but the photon left behind in the cavity is a marker indicating that the atom passed through it. The version of the quantum eraser in Scully *et al.* (1991) involves two such micromaser cavities which serve as the detectors yielding information about which slit an atom has traversed. By activating a photo-detector placed between the cavities, it is possible to erase this information, via the absorption of the information carrying photon.

Significantly however, detailed analysis reveals that such a device can detect the photon only half the time, at random, and it is only if the eraser actually detects the photon that the normal interference effects are to be expected. In fact, the situation is such that when the eraser works one gets the normal interference pattern, when it fails one gets an anti-interference pattern. These two patterns sum to the normal no-interference pattern as in our idealized example. Thus only if one already knows the state of the eraser can one detect the interference pattern, as a distinct pattern, on the distant screen. Perhaps one could say that such a device permits superluminal signalling in some very attenuated sense, but the receivers would not know what message had been sent until they got detailed records of the action of the eraser. Then they could correlate atom hits with eraser records and see which hits were parts of dots and which of dashes, but, of course, in such a case no usable information has been sent faster than light. In fact, it is this record of eraser operation which is the real message[6].

[6] Most quantum eraser experiments are performed with photons, exploiting our very advanced technological abilities in quantum optics. An interesting recent effort of this sort (Ma *et al.* (2013)) leads the authors to deny that quantum systems possess individual reality apart from their place within the totality of reality (at least that of system plus measuring device and the environment but in principle this could encompass the entire universe). If that is too grandiose a conclusion, the experiment amply verifies the mysterious universal non-local information level but non-causal connection between things.

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