There are many "standard" tests like the t-tests and analyses of variance that are commonly used. They rest on assumptions like normality, which can be hard to assess: for example, if you have small samples, how do you assess that the population(s) they came from are normal, when all you have to go on is your small samples? Also, the various kinds of analysis of variance require equal spreads of the observations within each group, and it is hard to be sure whether unequal sample variances mean that the population variances could be equal or not.

Historically, there was also a desire for tests where the calculation was easy (in the days before computers), so various so-called "non-parametric" tests were devised, which carried fewer assumptions than the standard tests. If the data really were normal, these tests would be less powerful than the standard ones, but if the data were sufficiently non-normal to make the standard tests invalid, the non-parametric tests would still be good. Many of these tests, instead of using the original data, rank the data values first in some way, then use the ranks, which obviously reduces the influence of outliers, and usually makes the data distributions less non-normal.

In many situations, there is a non-parametric test that corresponds to the standard test, as described below:

Situation	Standard test	Non-parametric test	
One sample, matched pairs	One-sample t-test	Sign test, signed rank test	
Two independent samples	Two-sample t-test	Rank sum test	
Two samples, comparing spread	F-test	Siegel-Tukey test	
One-way ANOVA	F-test	Kruskal-Wallis test, median test	
Randomized block design	F-test	Friedman test	
Ordered alternative in ANOVA	-	Terpstra-Jonckheere test	
Correlation	Pearson correlation	Spearman, Kendall correlation	
Regression	Regression	Iman-Conover regression	

Also, these are all tests, whereas we might be looking for confidence intervals. There is a standard procedure that can obtain a confidence interval from a test result, which can be applied to non-parametric tests as well.

Since we are using SAS, simplicity of computation is not an issue for us, but the easy-to-compute (by hand) tests often perform surprisingly well. Where possible, we will compare the standard test with the non-parametric one.

To start with single-sample tests for location: here are some data on time spent by orders in the administration department of a company (in working hours). Interest is in how the "typical" time compares to 8 hours.

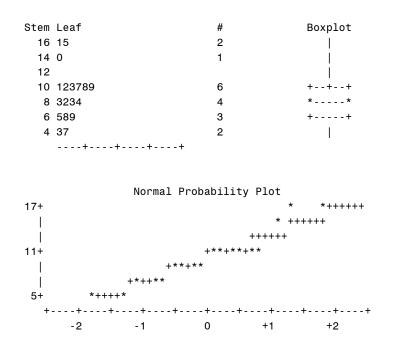
16.5 4.3 10.1 10.7 14.0 7.9 7.5 8.3 5.7 10.3 9.3 10.2 9.2 7.8 11.9 11.8 16.1 9.4

SAS's PROC UNIVARIATE does all of the one-sample tests, standard and non-parametric. We have to tell it that the null-hypothesis mean is 8, like this:

```
data admin;
infile "admin.dat";
input x @@;
```

```
proc univariate plot mu0=8;
```

The data values are all on one line, hence the @@. Looking at the plots first is probably helpful:



The data are mildly non-normal and seem skewed to the right, so maybe the median is a better measure of location than the mean. The sign test and signed-rank test are both testing the *median*, but the signed-rank test has the extra assumption that the distribution is symmetric, which is questionable here. The test results are:

Tests for Location: Mu0=8

Test	- S	tatistic-	p Valu	ıe
Student's t		2.727938	Pr > t	0.0143
Sign	М	4	Pr >= M	0.0963
Signed Rank	S	55	Pr >= S	0.0142

The t-test and the signed-rank test both give significant results (very similar P-values), but we have doubts about both normality and symmetry, so maybe we should be looking at the non-significant sign

test.

The sign test just looks at how many observations are above 8 (13) and below 8 (5), without considering how far above or below. The signed-rank test is essentially a t-test on the ranked data.

[The by-hand procedure: subtract the null-hypothesis mean from each observation, and rank the differences from smallest to largest *without considering whether they are plus or minus*. Then add up the ranks that correspond to the plusses or minuses, whichever is smaller.]

Matched-pair data (for example, before and after measurements on a number of subjects) are handled as above: in the DATA step, calculate the differences between the two measurements for each subject, and then test the null hypothesis that the mean (median) is zero for the differences.

Next, we have some data on the percentage of Japanese and British cars that break down in their first year (8 Japanese models and 12 British ones):

japan 3 japan 7 japan 15 japan 10 japan 4 japan 6 japan 4 japan 7 britain 19 britain 11 britain 36 britain 8 britain 25 britain 23 britain 38 britain 14 britain 17 britain 41 britain 25 britain 21

Are Japanese cars more reliable than British ones? The t-test is testing that the mean proportions are equal, but the test is ignoring that some of the British cars appear much less reliable than the rest (outliers). The rank sum test is based on this idea: if you pick a Japanese car and a British car at random, is it likely that the Japanese car's reliability will be better than the British one's (the answer appears to be "yes").

SAS has to do the two tests in two different PROCs. SAS's name for the rank sum test is "Wilcoxon":

```
data cars;
    infile "cars.dat";
    input type $ breakdowns;
proc ttest;
    class type;
    var breakdowns;
proc nparlway wilcoxon st;
```

```
class type;
var breakdowns;
```

(the ST thing I'll explain in a minute), with this output:

				Varia	ble: break	downs		
	type		Ν	Mean	Std Dev	Std Err	Minimum	Maximum
	brita	in	12	23.1667	10.5816	3.0546	8.0000	41.0000
	japan		8	7.0000	3.9279	1.3887	3.0000	15.0000
	Diff	(1-2)		16.1667	8.6270	3.9377		
type		Method		Mean	95% CL	Mean	Std Dev	95% CL Std Dev
britain				23.1667	16.4435	29.8899	10.5816	7.4959 17.9662
japan				7.0000	3.7162	10.2838	3.9279	2.5970 7.9944
Diff (1	-2)	Pooled		16.1667	7.8939	24.4394	8.6270	6.5187 12.7579
Diff (1	-2)	Sattert	hwaite	16.1667	9.0150	23.3184		
		Meth	od	Varianc	es D	0F t Value	Pr > t	
		Pool	ed	Equal	1	8 4.11	0.0007	
		Satt	erthwaite	e Unequal	15.00	9 4.82	0.0002	
				Equal	ity of Vari	ances		
			Method	Num DF	Den DF	F Value	Pr > F	
			Folded F	= 11	7	7.26	0.0148	

The TTEST Procedure

The equality of variances fails, so we should look at the Satterthwaite version of the test. The P-value is very small, so we'd reject the hypothesis that the means are equal.

The NPAR1WAY Procedure

Wilcoxon Scores (Rank Sums) for Variable breakdowns Classified by Variable type

type	Ν	Sum of Scores	Expected Under HO	Std Dev Under HO	Mean Score
japan	8	40.0	84.0	12.946855	5.000000
britain	12	170.0	126.0	12.946855	14.166667

Average scores were used for ties.

```
Wilcoxon Two-Sample Test

Statistic 40.0000

Normal Approximation

Z -3.3599

One-Sided Pr < Z 0.0004

Two-Sided Pr > |Z| 0.0008

t Approximation

One-Sided Pr < Z 0.0016

Two-Sided Pr > |Z| 0.0033

Z includes a continuity correction of 0.5.

Kruskal-Wallis Test

Chi-Square 11.5499

DF 1

Pr > Chi-Square 0.0007
```

Compare the "sum of scores" with "expected under H0" to see that the Japanese figures are on average quite a bit lower than the British ones. All the approximations say that the P-value is very small, so the null is rejected again.

In PROC TTEST, SAS gives two-sided P-values. We were doing a one-sided test (are the Japanese cars more reliable?), so we check that the sample means were the correct way around (they were) and then divide the P-value given by SAS, 0.0002, by 2. In PROC NPAR1WAY, the one-sided P-values are given; they are both small.

[By hand: rank all the data, then sum up the ranks of the values that came from the first sample, or the second sample, whichever is smaller.]

The Kruskal-Wallis test is used for one-way ANOVA situations, but just as you can also do ANOVA when comparing only 2 means, so you can use Kruskal-Wallis in the same situation. It gives a similar P-value here.

As we just saw, PROC TTEST does a 2-sample test for spread based on the sample variances. This could be questionable if variances are not the thing to use: if you have outliers, or if your population distributions are not normal. There is a non-parametric equivalent, which might be better for our car data (we were concerned about outliers), called the Siegel-Tukey test, which PROC NPAR1WAY will also do. (That's what the ST was.) The output looks like the rank sum test; in fact, the Siegel-Tukey test is just like the rank-sum test except that instead of assigning ranks to data values up from the smallest, it does so from the highest and lowest to the middle, so that if the two samples of the same size have the same spread, their Siegel-Tukey rank sums will be the same. (If they are not the same size, their rank *means* will be the same if the spreads are the same.)

The NPAR1WAY Procedure

	Siege		es for Variable by Variable †		
type	Ν	Sum of Scores	Expected Under HO	Std Dev Under HO	Mean Score
japan britain	8 12	74.0 136.0	84.0 126.0	12.868566 12.868566	9.250000 11.333333

Average scores were used for ties.

Siegel-Tukey Two-Sample Test

Statistic	74.0000
Z	-0.7382
One-Sided Pr < Z	0.2302
Two-Sided Pr > Z	0.4604

Z includes a continuity correction of 0.5.

Siegel-Tukey One-Way Analysis

Chi-Square	0.6039
DF	1
Pr > Chi-Square	0.4371

This test says that there is no evidence of a difference in spreads, in contrast to the F-test. This surprises me, because the British values are "obviously" more spread out, however you measure spread – for example, the interquartile ranges are 15 and 4.5, hardly equal.

PROC NPAR1WAY will also do standard and non-parametric one-way ANOVA. Another study of the reliability of cars, this time from 4 different countries, gave this:

britain 21 britain 33 britain 12 britain 28 britain 41 britain 39 britain 24 britain 29 britain 30 britain 19 britain 27 britain 38 britain 23 japan 9 japan 13 japan 6 japan 3 japan 5 japan 10 japan 4

germany 18 germany 35 germany 8 germany 17 germany 22 germany 20 germany 37 germany 11 italy 41 italy 41 italy 48 italy 34

The differences in location seem to be pretty obvious, but there are also differences in spread that we need to worry about: the British and German values seem more spread out than the others. We'll do a standard ANOVA as well as the Kruskal-Wallis test (called, confusingly, "wilcoxon" by PROC NPAR1WAY) and the Mood median test (called "median" in PROC NPAR1WAY; the "mood" option there is something else! Thus, this code:

```
data cars;
  infile "cars2.dat";
  input country $ percent;
proc npar1way anova wilcoxon median;
  var percent;
  class country;
```

produces this output:

The NPAR1WAY Procedure

		Classified b	y Variable co	•	
	с	ountry	Ν	Mean	
	b	ritain	13	28.000000	
	j	apan	7	7.142857	
	g	ermany	8	21.000000	
	i	taly	4	41.000000	
Source	DF	Sum of Squares	Mean Squa	ire F Value	Pr > F
Among	3	3410.017857	1136.6726	19 17.9523	<.0001
Within	28	1772.857143	63.3163	27	
	Wilco	xon Scores (Rank Classified b	Sums) for Va y Variable co	•	
		Sum of	Expected	Std Dev	Mean
ountry	Ν	Scores	Under HO	Under HO	Score

Analysis of Variance for Variable percent

country	N	Scores	Under HO	Under HO	Score
britain	13	262.0	214.50	26.052870	20.153846
japan	7	33.0	115.50	21.929368	4.714286
germany	8	117.0	132.00	22.969826	14.625000
italy	4	116.0	66.00	17.543494	29.000000

Average scores were used for ties.

Kruskal-Wallis Test

Chi-Square	20.4582
DF	3
Pr > Chi-Square	0.0001

The NPAR1WAY Procedure

Median Scores (Number of Points Above Median) for Variable percent Classified by Variable country

country	Ν	Sum of Scores	Expected Under HO	Std Dev Under HO	Mean Score
britain	13	10.0	6.50	1.411359	0.769231
japan	7	0.0	3.50	1.187977	0.00000
germany	8	2.0	4.00	1.244342	0.250000
italy	4	4.0	2.00	0.950382	1.000000

Average scores were used for ties.

Median One-Way Analysis

Chi-Square	16.2452
DF	3
Pr > Chi-Square	0.0010

The standard ANOVA produces a small P-value, but we should treat this with caution because of the unequal spreads within the groups. The Kruskal-Wallis test also produces a small P-value, which is more trustworthy because it is (essentially) a one-way ANOVA on the ranked data, which shouldn't be troubled by outliers.

Mood's median test is the easiest one to do by hand: work out the overall median (of all the data), and count how many values are above and below the median in each group. If the groups are all about the same, the observations should be about 50-50 above and below the overall median in each group. Here the overall median is 22.5; all the Japan figures are below this, and all the Italy figures are above. The counts of below-median and above-median for each country form a two-way table, which is then analyzed using a chi-squared test. Mood's median test is a lot like the sign test generalized to two or more groups.

In the one-way ANOVA situation, you might know the order in which the groups should lie, if the null hypothesis is not true. A test that assesses the evidence against the null, while allowing for the alternative order, should be more powerful (should give you a greater chance of rejecting the null in favour of that specific alternative) than, say, the Kruskal-Wallis test. Such a test is the Jonckheere-Terpstra test; it is really a rank sum test done on each pair of groups, so that if the ordering is as claimed, there will be a bunch of small rank sum test statistics to add together.

Some data: these are lifetimes of light bulbs of three different brands. Brand A is bought in a discount store, brand B in a grocery store, and brand C in a hardware store. We might expect brand C lightbulbs to last longer on average than brand B, which in turn last longer than brand A. We can test an alternative that the lifetimes go A, B, C in that order.

a 619 a 35 a 126 a 2031 a 215 b 343 b 2437 b 409 b 267 b 1953 b 1804 c 3670 c 2860 c 502 c 2008 c 5004 c 4782

These lifetimes are obviously not normal (lifetimes of things are usually skewed to the right) so that even if we had a "standard" test to use against the ordered alternative, it wouldn't apply here, and we would want to use something non-parametric, which the Jonckheere-Terpstra test is.

SAS doesn't make it that easy to do this test: it is hiding as an option of the TABLES command in PROC FREQ (which is usually for making contingency tables, doing chi-squared tests and the like). Also, it is a good idea to arrange your labels (here a, b, c) in the order that they appear in the alternative hypothesis (SAS can understand which order you want, but I wasn't able to get it working reliably).

Here is my code:

```
data light;
    infile "lightbulbs.dat";
    input brand $ lifetime;
proc freq;
    tables brand*lifetime / jt;
```

Never mind that "lifetime" isn't in any sense a categorical variable, so that making a contingency table of it with "brand" seems kind of pointless. Here's what you get (which is SAS's best effort at making a contingency table anyway):

brand lifetime

Frequency Percent

ROW PCT										
Col Pct	35	126	215	267	343	409	502	619	1804	Total
a	1	1	1	0	0	0	0	1	0	5
	5.88	5.88	5.88	0.00	0.00	0.00	0.00	5.88	0.00	29.41
	20.00	20.00	20.00	0.00	0.00	0.00	0.00	20.00	0.00	
	100.00	100.00	100.00	0.00	0.00	0.00	0.00	100.00	0.00	
b	0	0	0	1	1	1	0	0	1	6
	0.00	0.00	0.00	5.88	5.88	5.88	0.00	0.00	5.88	35.29
	0.00	0.00	0.00	16.67	16.67	16.67	0.00	0.00	16.67	
	0.00	0.00	0.00	100.00	100.00	100.00	0.00	0.00	100.00	
с	0	0	0	0	0	0	1	0	0	6
	0.00	0.00	0.00	0.00	0.00	0.00	5.88	0.00	0.00	35.29
	0.00	0.00	0.00	0.00	0.00	0.00	16.67	0.00	0.00	
	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	
Total	1	1	1	1	1	1	1	1	1	† 17
	5.88	5.88	5.88	5.88	5.88	5.88	5.88	5.88	5.88	100.00
(0 +	-15									

(Continued)

Table of brand by lifetime

brand lifetime

Frequency

Percent Row Pct

Row Pct									
Col Pct	1953	2008	2031	2437	2860	3670	4782	5004	Total
a	0	0	1	0	0	0	0	0	- 5
	0.00	0.00	5.88	0.00	0.00	0.00	0.00	0.00	29.41
	0.00	0.00	20.00	0.00	0.00	0.00	0.00	0.00	
	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	
b	1	0	0	1	0	0	0	0	- 6
	5.88	0.00	0.00	5.88	0.00	0.00	0.00	0.00	35.29
	16.67	0.00	0.00	16.67	0.00	0.00	0.00	0.00	
	100.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	
c	0	1	0	0	1	1	1	1	- 6
	0.00	5.88	0.00	0.00	5.88	5.88	5.88	5.88	35.29
	0.00	16.67	0.00	0.00	16.67	16.67	16.67	16.67	
	0.00	100.00	0.00	0.00	100.00	100.00	100.00	100.00	
Total	1	1	1	1	1	1	1	 1	- 17
	5.88	5.88	5.88	5.88	5.88	5.88	5.88	5.88	100.00

Statistics for Table of brand by lifetime

Jonckheere-Terpst	ra Test
Statistic	81.0000
Z	2.9055
One-sided Pr > Z	0.0018
Two-sided Pr > Z	0.0037
Sample Size	= 17

The table suggests that most of the short lifetimes went with Brand A and most of the long ones with Brand C, which is confirmed by the test: the P-value is very small, so we reject a null hypothesis that the "average" lifetimes are equal in favour of the alternative A, B, C.

You could do an ordinary ANOVA here, like this, with some trepidation:

```
proc glm;
    class brand;
    model lifetime=brand;
```

with the following output:

The GLM Procedure

Dependent Variable: lifetime

			Sum of			
Source		DF	Squares	Mean Square	F Value	Pr > F
Model		2	19879674.56	9939837.28	6.27	0.0114
Error		14	22203092.97	1585935.21		
Corrected T	otal	16	42082767.53			
	R-Square	Coeff	Var Root M	ASE lifetime	Mean	
	0.472395	73.6	5824 1259.3	339 1709	.706	
Source		DF	Type I SS	Mean Square	F Value	Pr > F
brand		2	19879674.56	9939837.28	6.27	0.0114
Source		DF	Type III SS	Mean Square	F Value	Pr > F
brand		2	19879674.56	9939837.28	6.27	0.0114

This P-value of 0.0114 is bigger than we had from Jonckheere-Terpstra, for a couple of reasons: one, we shouldn't really have done an ordinary ANOVA here (the data are far from normal), and two, the ANOVA alternative is just "the means are not all the same", which is not really the alternative that we want: we'd have to go on and do multiple comparisons to verify that the means came out in the right order.

The next step beyond one-way ANOVA is a randomized blocks design. An example: we want to see whether people's pulse measurements vary during the day. We have three patients available, and we measure each one's pulse at 6 and 10 am and pm. We could just think of the three patients as replicates and do a one-way ANOVA (or Kruskal-Wallis), but some people have "typically" different pulse rates than others, which means that an observed pulse rate could depend both on time of day and patient. This can be handled by treating patients as "blocks", which means we are taking it for granted that patients will be different, and allowing for patient differences in assessing the time-of-day effect (like including a second variable in a multiple regression not because we care about it, but because we want to allow for its effect).

In the non-parametric world, we can use Friedman's test (named after Milton Friedman the economist, who was apparently doing some moonlighting). The idea is to use ranks *within* blocks. Here's some data:

Patient Time K F R 0600 60 62 55 1000 65 66 65 1800 70 80 75 2200 65 72 68

Patient F had the highest pulse at all the times, but we don't really care about that: what appears to be happening is that pulse rates rise to a maximum at 6 pm and then drop. Ranking within columns makes this clear:

	Patient						
Time	Κ	F	R				
0600	1	1	1				
1000	2.5	2	2				
1800	4	4	4				
2200	2.5	3	3				

Even though the actual pulse rates are not the same, the rankings over time of day are almost identical, and certainly way more similar than you'd expect by chance.

SAS claims to be able to do the Friedman test, but it is a very obscure process, and I prefer to construct a "home-brew" test by obtaining the ranks and doing a randomized-blocks ANOVA (a two-way ANOVA without interaction) on the ranks.

Here are the data for SAS purposes:

We first need to obtain the ranks of the pulse rates within each patient, and to do *that* we need the data sorted by patient first. Once we've done that, we run the ANOVA on the ranks using PROC GLM. Here's all the code:

```
data pulse;
    infile "pulse.dat";
    input time $ patient $ pulse;
proc sort;
    by patient;
proc rank;
    by patient;
    var pulse;
    ranks rpulse;
proc print;
proc glm;
    class patient time;
    model rpulse=time patient;
```

PROC RANK produces a new data set that contains the ranked pulse values within each patient, with the stored ranks in the variable "rpulse". I then do a PROC PRINT so that you can see what the output of PROC RANK looks like, and then I run it through PROC GLM using the ranked-within-patient pulse rates as the response.

Here is the output from PROC RANK:

0bs	time	patient	pulse	rpulse
1	0600	F	62	1.0
2	1000	F	66	2.0
3	1800	F	80	4.0
4	2200	F	72	3.0
5	0600	К	60	1.0
6	1000	К	65	2.5
7	1800	К	70	4.0
8	2200	K	65	2.5
9	0600	R	55	1.0
10	1000	R	65	2.0
11	1800	R	75	4.0
12	2200	R	68	3.0

where you can compare the ranked pulse values for each patient with the table above. (Patient K had two values 65, which share the 2nd and 3rd ranks.) The ranks are very consistent over time, which should show up in the ANOVA of ranks:

The GLM Procedure

Dependent Variable: rpulse Rank for Variable pulse

Source		DF	Sum Squai		Mean So	uare F	Value	Pr > F
Model		5	14.166666	667	2.8333	3333	51.00	<.0001
Error		6	0.333333	333	0.0555	5556		
Corrected Total		11	14.500000	000				
	R-Square	Coeff	Var	Root MS	SE rp	ulse Mean		
	0.977011	9.428	8090	0.23570)2	2.500000		
Source		DF	Type I	SS	Mean Sq	uare F	Value	Pr > F
time		3	14.166666	667	4.7222	2222	85.00	<.0001
patient		2	0.00000	000	0.0000	0000	0.00	1.0000
Source		DF	Type III	SS	Mean So	uare F	Value	Pr > F
time patient		3 2	14.16666 0.00000		4.7222 0.0000		85.00 0.00	<.0001 1.0000

as indeed it does. The P-value for a difference due to times is as small as it could be. (We organized the ranks so that ranks 1, 2, 3, 4 appear once for each patient, apart from the tied values, so that even if there is a patient effect with the original data, there isn't with the ranks.)

If you hadn't allowed for differences between patients, you would have done Kruskal-Wallis and gotten this:

	Wilco	,	Scores (Rank Sums) for Variable pulse Classified by Variable time							
		Sum of	Expected	Std Dev	Mean					
time	Ν	Scores	Under HO	Under HO	Score					
0600	3	6.0	19.50	5.370373	2.000000					
1000	3	17.0	19.50	5.370373	5.666667					
1800	3	32.0	19.50	5.370373	10.666667					
2200	3	23.0	19.50	5.370373	7.666667					

The NPAR1WAY Procedure

Average scores were used for ties.

Kruskal-Wallis Test

Chi-Square	9.2837
DF	3
Pr > Chi-Square	0.0257

for which the P-value is also small. The P-value from Friedman's test is smaller, because it allows for differences due to patients as well. (In less fortunate circumstances, the Kruskal-Wallis test might not have been significant at all because the difference between patients might have overwhelmed the difference due to time of day.)

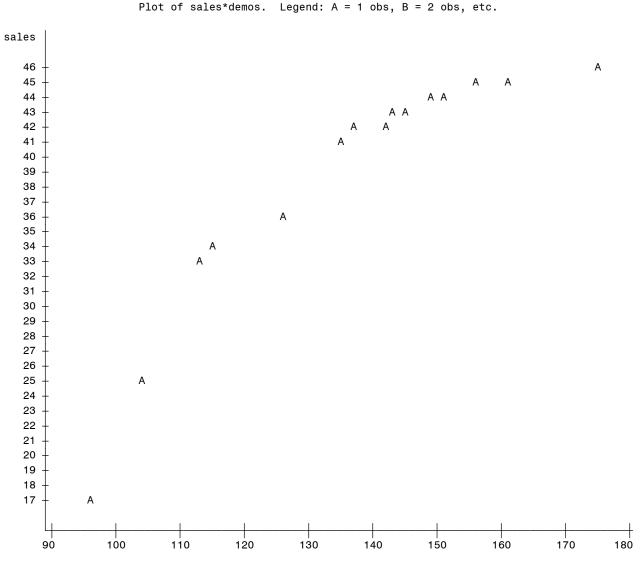
[There aren't specific non-parametric tests for two-way ANOVA and above, but you can always replace the data by their ranks and see whether it makes any difference. You rank all the data, and then look to see whether the mean ranks are significantly different.]

Our final procedure is correlation. There is the usual "Pearson" correlation, but there are also two kinds of non-parametric correlation. SAS PROC CORR handles them all. As an example: a company is trying to see what its sales staff can do to increase sales. One kind of visit a sales rep can do is a "demo", where the sales rep goes to a potential customer and demonstrates how the product works (and answers questions about it). A record is kept of the number of demos and number of sales for each sales rep. The data look like this, with demos in the first column and sales in the second:

The first step with this kind of data is to draw a scatterplot to get a sense of the relationship. Then we have SAS calculate the correlations, the usual one called "Pearson", and the two non-parametric ones called "Spearman" and "Kendall". The code looks like this:

```
data demos;
    infile "demos.dat";
    input demos sales;
proc plot;
    plot sales*demos;
proc corr pearson spearman kendall noprob;
    var sales demos;
```

SAS by default does a test that each correlation is zero. A look at the plot reveals that there is obviously a relationship, so we skip the tests (with "noprob"):





The trend is clearly upward, but it appears to level off about about 140 demos, so there is a "diminishing returns" thing happening here. Recall that the usual Pearson correlation is only good for straight-line relationships, so we shouldn't trust it here. However, the Spearman and Kendall correlations are good for any "monotonic" relationship: one that keeps going up (like this one) or keeps going down, even if the trend is a curve.

The Spearman correlation uses the ranked demos and sales; it is the ordinary correlation but of the ranks. So it will be high if a low rank on one variable goes with a low rank on the other, and high with high: here, the smallest number of demos goes with the smallest number of sales, and the largest with the largest, so the Spearman correlation will be high even though the trend is curved.

The Kendall correlation is even "more non-parametric" than the Spearman one: take a pair of points, say the two at the bottom left. For the one where the demos is higher, is the number of sales higher? Here, the answer is "yes"; this pair is called "concordant". This is then repeated for all pairs of points, and here almost all of them are concordant, so the Kendall correlation should also be close to 1:

The CORR Procedure

2 Variables: sales demos

Simple Statistics

Variable	Ν	Mean	Std Dev	Median	Minimum	Maximum
sales demos	15 15	38.66667 136.53333	8.33809 22.03525	42.00000 142.00000	17.00000 96.00000	46.00000 175.00000
		Pearson Corr	elation Coeffi	icients, N = 15		
			sales	demos		
		sales	1.00000	0.92124		
		demos	0.92124	1.00000		
		Spearman Corr	elation Coeffi	icients, N = 15		
			sales	demos		
		sales	1.00000	0.99642		
		demos	0.99642	1.00000		
	ĸ	Cendall Tau b C	orrelation Coe	efficients, N =	15	
			sales	demos		
		sales	1.00000	0.98077		
		demos	0.98077	1.00000		

The correlations are: Pearson 0.921, Spearman 0.996, Kendall 0.981. The two non-parametric ones are higher because the trend isn't a straight line. (Spearman usually comes out closer to 1 than Kendall does).

SAS does tests that all 3 correlations are zero; for Pearson and Spearman, you can also test a specific value in the null, say 0.9, like this:

```
proc corr pearson spearman fisher(rho0=0.9);
  var sales demos;
```

with this output:

Pearson Correlation Statistics (Fisher's z Transformation)									
Variable	With Varial	ble	N Corre	Sample elation	Fisher's z	Bias Adjustment			
sales	demos		15 (.92124	1.59718	0.03290	0.91611		
	Pe	arson Correla	tion Statis	stics (Fish	ner's z Trans	sformation)			
With Variable Variable 95% Confidence Limits						H0 : Rho=Rh Rho0	p Value		
sa	les	demos	0.7609	959 0.	972153	0.90000	0.7478		
			The (CORR Proced	lure				
	Sp	earman Correl	ation Stat:	istics (Fis	sher's z Tra	nsformation)			
Variable	With Varial	ble	N Corre	Sample elation	Fisher's z	Bias Adjustment			
sales	demos		15 (.99642	3.16218	0.03559	0.99616		
	Spearman Correlation Statistics (Fisher's z Transformation)								
Va	riable	With Variable	95% Cor	nfidence Li		H0 : Rho=Rh Rho0	p Value		
sa	les	demos	0.988	138 0.	998760	0.90000	<.0001		

Look at the second of each pair of lines. The Pearson correlation is not significantly different from 0.9, and the 95% confidence interval includes 0.9, but the Spearman correlation *is* significantly different from 0.9, and the confidence interval shows that it is significantly higher.