Questions

1. Give some examples of random variables associated with tossing coins and rolling dice. In each case, what else do you need to complete the specification of a probability distribution?

   – *tossing 2 coins, counting heads*
   – *rolling 2 dice, total spots*
   – *rolling 2 dice, number of 6's*
   – *tossing 10 coins, 2 points for H and 1 point for T*

Point: each outcome gives ***number****.*

2. For this probability distribution:

Value of X	1	2	3
Probability	0.3	0.6	0.1

(a) Find $P(X \geq 2)$.
(b) Find the probability that X is either 1, 2 or 3. Does your answer make sense?

a: P(X>=2)=P(X=2 or 3)
=P(X=2)+P(X=3)=0.6+0.1=0.7
b: P(X=1 or 2 or 3)=P(X=1)+P(X=2)+P(X=3)=1
One of 1, 2 or 3 must be observed

3. Toss 3 fair coins, count # heads (X):

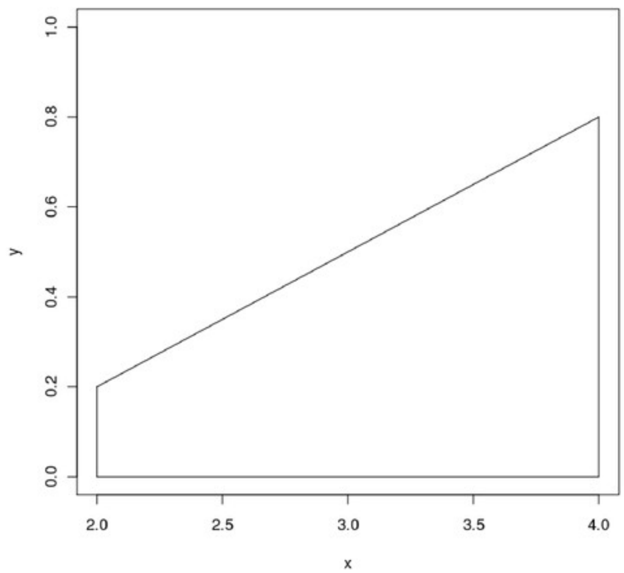
   (a) Write down all 8 possible outcomes, the number of heads contained in each, and the probability of each.
   (b) Find $P(X = 2)$. What is this the probability of, in words?
   (c) Make a table of the probability distribution of X.

HHH 3
HHT 2
HTH 2
HTT 1
THH 2
THT 1
TTH 1
TTT 0 each prob 1/8
P(X=2)=P(HHT or HTH or THH)=3/8

Value	0	1	2	3
Prob	{1/8}	{3/8}	{3/8}	{1/8}

This random variable **discrete** (separated values 0, 1, 2, 3 (eg. 1.5 impossible)

4. A continuous random variable X has a density function that is the shape of a trapezoid, as shown below. The area of a trapezoid is its base times the average of the heights at the two ends.



(a) Verify that this is indeed a density function.

(b) Would you guess that $P(X > 3)$ is more or less than 0.5? Explain.

(c) The density function at $x = 3$ is 0.5. Find $P(X > 3)$. (Drawing a picture might be helpful.) Was your guess correct?

a: is area under density 1?
Area = (4-2)(0.2+0.8)/2 = 1 ok
b: values bigger than 3 more likely than values less than 3, so guess P(X>3) more than 0.5.
c: P(X>3) area under density between 3 and 4:
(4-3)(0.5+0.8)/2=0.65 (b was good)

5. For this distribution:

Value of X	1	2	3
Probability	0.3	0.6	0.1

find the mean of X. Does it make sense that the mean is less than 2?

Mean=value x prob added up
=(1)(0.3)+(2)(0.6)+(3)(0.1)=1.8
less than 2 because X=1 more likely than X=3

6. The distribution of number of heads in 3 tosses of fair coin is:

Value of X	0	1	2	3
Probability	1/8	3/8	3/8	1/8

(a) What is the mean number of heads?
(b) Is this answer surprising?

a: (0)(1/8)+(1)(3/8)+(2)(3/8)+3(1/8)=1.5
b: not at all: expect 50% of 3 = 1.5 heads

7. Previously, we investigated the Poisson distribution with mean 3 as a model for goalscoring in hockey games. If we take samples from this distribution, how close might the sample mean be to 3? Use StatCrunch as below:
    (a) Generate a population of 1000 Poisson values by using Data, Simulate, Poisson with 1000 rows and 1 column.
    (b) Take 100 samples of size 5 from this population, using Data, Sample from Columns. Check "Stacked with Sample ID".
    (c) Calculate the mean for each sample using Stat, Summary Stats, and group by Sample, saving the results in a data table.
    (d) Make a stemplot of the sample means. Are they usually close to 3.
    (e) Repeat (b), (c) and (d) for samples of size 100.
    (f) Is the mean of a bigger sample usually closer to 3? What does that say about the informativeness of larger samples compared with smaller ones?

See my StatCrunch report
http://www.statcrunch.com/5.0/viewreport.php?reportid=22403
for this one.

8. This distribution has mean 1:

Value of X	0	1	2
Probability	0.4	0.5	0.1

Find the variance and thus the standard deviation of X.

Error: should be

Value	0	1	**5**
Prob	0.4	0.5	0.1

Variance:
(0-1)^2(0.4)+(1-1)^2(0.5)+(5-1)^2(0.1)
= 0.4 + 0 + 1.6 = 2
so SD = sqrt(2)=1.414

9. A random variable X has this distribution, with mean 0.9 and SD 0.54:

Value	0	1	2
Prob.	0.2	0.7	0.1

The distribution of the random variable $3X$ is obtained by multiplying all the values by 3 and leaving the probabilities unchanged.

(a) Write down the distribution of $3X$.
(b) Calculate the mean of $3X$. How does it compare to the mean of X?
(c) Calculate the SD of $3X$. How does it compare to the SD of X?

a:

value	0	3	6
prob	0.2	0.7	0.1

b: mean (0)(0.2)+3(0.7)+6(0.1)=2.7 (3 x bigger)
c: variance (0-2.7)^2(0.2)+(3-2.7)^2(0.7)+(6-2.7)^2(0.1)=2.610, sd=sqrt(2.610)=1.616 (also 3 x bigger)

10 A random variable X has this distribution, with mean 0.9 and SD 0.54:

Value	0	1	2
Prob.	0.2	0.7	0.1

The distribution of the random variable $X + 4$ is obtained by adding 4 to all the values and leaving the probabilities unchanged.

(a) Write down the distribution of $X + 4$.
(b) Calculate the mean of $X + 4$. How does it compare to the mean of X?
(c) Calculate the SD of $X + 4$. How does it compare to the SD of X?

a:

value	4	5	6
prob	0.2	0.7	0.1

b: mean (4)(0.2)+(5)(0.7)+(6)(0.1)=4.9
(4 bigger)
c: variance: (4-4.9)^2(0.2)+(5-4.9)^2(0.7)
+(6-4.9)^2(0.1)=0.290;
sd=sqrt(0.290)=0.54 (same)

11. X takes values 0 and 1 with $P(X = 0) = 0.3, P(X = 1) = 0.7$. Independently of X, Y takes values 0 and 1 with $P(Y = 0) = 0.4, P(Y = 1) = 0.6$. X has variance 0.21 and Y has variance 0.24. Let $Z = X + Y$.
    (a) Verify that $P(Z = 0) = 0.12$, $P(Z = 1) = 0.46$, $P(Z = 2) = 0.42$.
    (b) Z has mean 1.3. What is the variance of Z?
    (c) How could you have worked out the variance of Z without working out the three probabilities for Z?
    (d) What if you didn't know that X and Y were independent?

a: P(Z=0)=P(X=0 and Y=0)=P(X=0) x P(Y=0)
= 0.3 x 0.4 =0.12
P(Z=1)=P((X=0 and Y=1) or (X=1 and Y=0))
=P(X=0)P(Y=1)+P(X=1)P(Y=0)
=0.3 x 0.6 + 0.7 x 0.4 = 0.46

b: variance of Z: (0-1.3)^2(0.12)+
(1-1.3)^2(0.46)+(2-1.3)^2(0.42)=0.45

ie. means add (and subtract)
and **variances** add if random variables are independent

that is: mean of A+B is mean of A plus mean of B
mean of A-B is mean of A minus mean of B
if A and B are independent:
variance of A+B is variance of A plus variance of B;

variance of A-B is variance of A *plus* variance of B (see summary of Chap 4)