Questions

1. Suppose population 1 has mean μ_1, pop. SD σ_1 and population 2 has mean μ_2, SD σ_2. Samples are taken from each population: the sample from population 1 has size n_1, sample mean $\bar{x}_1$, sample SD s_1, and the sample from population 2 has size n_2, sample mean $\bar{x}_2$, sample SD s_2. Assume that both populations are normal.
   (a) What is the sampling distribution of $\bar{x}_1$? $\bar{x}_2$?
   (b) What is the sampling distribution of $\bar{x}_1 - \bar{x}_2$?
   (c) What therefore is the distribution of $\bar{x}_1 - \bar{x}_2$, suitably standardized?
   (d) In practice σ_1^2 and σ_2^2 are not known, and are replaced by s_1^2 and s_2^2. What distribution would you guess the resulting statistic to have?

(read this later)

a: normal with mean mu1, SD sigma1/sqrt(n1), and likewise mean mu2, SD sigma2/sqrt(n2).
b: normal with mean mu1-mu2, variance sigma1^2/n1+sigma2^2/n2, SD square root of this.
c: normal (as long as samples large enough)
d: t.

2. Using the results of the previous question, write down the formulas for the two-sample t confidence interval and test statistic.

CI: $\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

test stat: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

base df on *smaller* sample size

3. A study assessed the effect of piano lessons on spatiotemporal reasoning.
Treatment group: 34 preschool children given piano lessons for 6 months, spatiotemporal reasoning measured.
Control group: 44 preschool children with no piano lessons, spatiotemporal reasoning measured at end.
Data:

Group	n	Mean	SD
Treatment	34	3.62	3.06
Control	44	0.39	2.42

(a) Calculate the standard error of the difference in sample means.
(b) Obtain a 95% confidence interval for the difference in population means. Do the piano lessons appear to be effective?
(c) Is there evidence of a difference in mean spatiotemporal reasoning score between the two groups? Do a suitable test.
(d) Use StatCrunch to do the last two parts, and compare with the answers you obtained. Why is there a difference?

a: sqrt(3.06^2/34+2.42^2/44)=0.6391
b: t*=2.042 (correct df 33, use 30)
3.62-0.39 +/- 2.042(0.6391) = 1.92 to 4.54
c: t=(3.62-0.39)/0.6391=5.05; use 30 df to get P-value < 0.0005
d: StatCrunch uses “software df” to get more accurate answer. For hand calculation, use df as above.

4. A study of native butter clams measured the width and length of 4 randomly chosen clams:

Width	3.1	4.2	4.8	5.6
Length	4.1	5.5	6.6	7.0

(a) Is a paired or two-sample t-test more appropriate? Explain.

(b) The four differences, width minus length, have sample mean -1.375 and sample SD 0.33. Does this information enable you to calculate a 95% confidence interval for width minus length for all clams? If so, do it.

(c) Enter the data into StatCrunch and do the appropriate test. If you have results to compare, compare them.

a: matched pairs

b: t*=3.182 (3 df); ci -1.375 +/- 3.182(0.33/sqrt(4))=-1.90 to -0.85

width is significantly less than length

5. A comparison was made between the numbers of cigarettes smoked per day by randomly chosen females and males. The 33 females had a sample mean of 6.94 and sample SD of 7.47; the 19 males had a sample mean of 4.21 and sample SD of 5.57.
   (a) Is a paired or two-sample t test more appropriate? Explain.
   (b) Use the data, if possible, to assess the evidence that the mean numbers of cigarettes smoked per day is different for males and females. What do you conclude?
   (c) Do your analysis in StatCrunch, explaining differences from (b), if any.

Two-sample (no pairing).
Ha: mu1 not equal to mu2 (2-sided)
SE=sqrt(7.47^2/33+5.57^2/19)=1.8231
t=(6.94-4.21)/1.8231=1.497
P-value (18 df) between 2 x 0.05 and 2 x 0.10, ie. 0.10 and 0.20. No evidence of difference in means.

6. In the situations below, would you be happy to use a two-sample t procedure, or not? If not, why not?
   (a) $n_1 = 50, n_2 = 30$, populations both normal.
   (b) $n_1 = 50, n_2 = 30$, populations mildly non-normal.
   (c) $n_1 = 80, n_2 = 2$, populations might not be normal.
   (d) $n_1 = 20, n_2 = 20$, population 1 skewed to left, population 2 skewed to right.
   (e) $n_1 = 20, n_2 = 20$, both populations skewed to right.

a and b ok
c: trouble because of very small sample
d: might be ok (skewness might cancel out)
e: difference still skewed to right (no good)

*** WE ARE DONE!!!! *****