Questions

1. Explain how the Canadian legal system tries to judge the innocence or guilt of an accused person by assessing the evidence presented in court.
2. Relate the above to judging whether a statement about a population mean ("null hypothesis") is true or false, based on evidence contained in sample mean.

	Decision	
Truth	Guilty	not guilty
Guilty	correct	error
Innocent	bad error	correct

	Decision for H0	
Truth about H0	Rejected	Not rejected
false	correct	type II error
true	type I error	correct

3. Calcium levels in the blood of healthy young adults vary with mean 9.5 and SD 0.4 mg/dl. Suppose we conduct a study of pregnant women in rural Guatemala: is their blood calcium level different on average?
   (a) What are we trying to prove here? This is the alternative hypothesis. Write it in symbols.
   (b) If the pregnant women are in fact no different from young adults generally, what would be true about their mean calcium blood level? This is the null hypothesis. Write it in symbols.
   (c) Suppose we had been trying to prove that these women had a *higher* blood calcium level than healthy young adults generally. What would the alternative hypothesis have been then?

a: trying to prove difference from 9.5: $H_a: \mu \neq 9.5$
b: no difference, mean is 9.5: $H_0: \mu = 9.5$
c: would be $H_a: \mu > 9.5$

4. In the previous question, suppose the null hypothesis is true, and we intend to take a sample of 180 pregnant women.
   (a) What is the sampling distribution of the sample mean in that case?
   (b) The observed sample mean is 9.58. How likely are we to get a value this far or farther from 9.5, if the null hypothesis is true? (This is using $H_a : \mu \neq 9.5$). Your result is called the P-value for this test.
   (c) On the basis of this sample, do you think the population mean for Guatemalan pregnant women is 9.5, or not? Explain briefly.
   (d) How would your P-value change if we had $H_a : \mu > 9.5$?
   (e) How would your P-value change if we had $H_a : \mu < 9.5$?

a & b: calculate $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{9.58 - 9.5}{0.4 / \sqrt{180}} = 2.68$.

P(above)= 0.0037, so P(this far or farther from 9.5)=2 x 0.0037 = 0.0074.

Because this is very small, believe that mu is not 9.5 after all: reject H0.

d: prob of (>2.68) = 0.0037

e: prob of (<2.68) = 0.9963

5. What do you conclude in the following cases:
   (a) $\alpha = 0.05$, P-value 0.027.
   (b) $\alpha = 0.01$, P-value 0.027.
   (c) $\alpha = 0.10$, P-value 0.027.
   (d) $\alpha = 0.01$, P-value 0.007.
   (e) $\alpha = 0.05$, P-value 0.45.

Choose alpha first, then compare P-value with alpha. If P-value is smaller, reject H0; if not, do not reject H0.
a,c,d: reject H0
b, e: do not reject H0

6. In what kind of situation might you choose:
   (a) a small α like 0.01?
   (b) a large α like 0.10?

a: if rejecting H0 is important/expensive
b: exploring/pilot study

default alpha=0.05

7. The General Health Questionnaire (GHQ) measures mental health (low score better). In general population, SD is $\sigma = 5$. Researcher wants to show that mean GHQ for all unemployed men exceeds 10. Sample of 49 unemployed men, sample mean 10.94.
   - (a) Write down suitable null and alternative hypotheses.
   - (b) Choose a value for α.
   - (c) Calculate the test statistic and P-value for your hypotheses. What do you conclude?
   - (d) Check your calculations using StatCrunch.

Let μ be population mean GHQ for all unemployed men.

Ha: mu>10; H0: mu=10
alpha=0.05
z=(10.94-10)/(5/sqrt(49))=1.32
P-value = P(>1.32) = 0.0941
do not reject H0, no evidence that population mean for unemployed men is > 10

Ken's notes: z=1.32, P=0.0941

8. Summarize how you would calculate P-values for each combination of: the possible alternative hypothesis, and whether $z > 0$ or $z < 0$. Draw pictures in each case.

Suppose $H_0: \mu = \mu_0$:

	$z < 0$	$z > 0$
$H_a: \mu \neq \mu_0$	2 x prob of < z	2 x prob of > z
$H_a: \mu < \mu_0$	prob of < z	prob of < z (large: wrong side)
$H_a: \mu > \mu_0$	prob of > z (large: wrong side)	prob of > z

9. A sample of 24 male long-distance runners gave a sample mean weight of 61.8 kg. Assume that the SD of the weights of all male long-distance runners is 4.5 kg. Let μ be the mean weight of all male long-distance runners. Use StatCrunch for the following:
   (a) Calculate a 95% CI for μ.
   (b) Calculate a 99% CI for μ.
   (c) Obtain P-values for tests of the following H_0 values of μ, against a two-sided alternative: 59, 62, 64.
   (d) Display all your results in a table, and explain how your tests and confidence interval results are compatible.

a: 60.0 to 63.6; b: 59.4 to 64.2
c: 0.0023; 0.8276; 0.0166

μ	P-value	reject at 0.05?	inside 95% CI?		reject at 0.01?	inside 99% CI?
59	0.0023	yes	no		yes	no
62	0.8276	no	yes		no	yes
64	0.0166	yes	no		no	yes

Also: read lecture 21 notes, questions 1-5 (also on website).