Why does Proof by Induction Work?
A Contradiction Argument

We need

The Well Ordering Principle:

Any nonempty subset C of $\mathbb{N}$ contains a smallest element.

We say that C is *well ordered*.

$$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \ldots i-1\ i\ i{+}1\ i{+}2\ \ldots n-1\ n\ n{+}1\ \ldots$$

Suppose $C = \{3, 7, 9, 10, i, n, \ldots\}$ then 3 is the smallest element.

Assume that we have done the 3 steps of an *inductive proof* properly.

1.
2.
3.

We now construct a *contradiction*.

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This is where the *Well Ordering Principle* comes into play.

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Therefore, our *assumption must be false*, and $P(n)$ holds for all $n \in \mathbb{N}$.

Example.

Prove using the **Well Ordering Principle** that:

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

for all positive integers n.

Proof.

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