CSCB36 WORKSHEET – WEEK 1 – PROOF BY INDUCTION

1 Proof by Induction

Lemma 1. *Prove that for all natural numbers n,*

$$12^n - 1 \textit{ is an integer} \text{ multiple } \textit{of } 11.$$

Proof.

1. **Define** $P(n)$:

   RTP: $P(n)$ is true $\forall n \in \mathbb{N}$.

2. **Base Case:**

3. **Inductive Hypothesis:**

4. **Inductive Step:**

□

Splitting Piles Consider this "*trick*". Start with a pile of n stones. Ask your friend to split the piles into two smaller piles of any size of at least 1. Multiply the sizes of the two piles and add to a sum that we will call $total$. Repeat until all piles are of size 1.
Example.

Let $n = 6$. Try splitting the pile in different ways and seeing what your total is. Here is one way:
$6 \to \frac{4}{2}$ makes a sum of 8. $total = 8$.
$4 \to \frac{3}{1}$ makes a sum of 3. $total = 11$.
$2 \to \frac{1}{1}$ makes a sum of 1. $total = 12$.
$3 \to \frac{1}{2}$ makes a sum of 2. $total = 14$.
$2 \to \frac{1}{1}$ makes a sum of 1. $total = 15$.

Try another way of splitting the piles with $n = 6$.

Try with $n = 8$ and $n = 9$. What do you notice about the $total$?

Let's prove the claim.

$P(n)$:

Base Case.

Induction Hypothesis.

Induction Step.