Non-Deterministic Finite State Automata (NFA or NFSA)

Q: What does *deterministic* mean?

the path is fixed → no choice.

NFSA. A *non-deterministic finite state automata* (*NFSA*) extends *DFSA* by *allowing* choice at each state.

Differences between *DFSA* and *NFSA*:

- **NFSA.** Given a state q_i and an input x there can be *more* than *one* possible *transition*, i.e,

$$\delta^*(q, x) = \{set\ of\ q_i\}$$

- **NFSA.** Given state q_i, we can have an ϵ *transition*.

$$\delta^*(q_i, \epsilon) = q_j$$

  This means we can spontaneously *jump* from q_i to q_j.

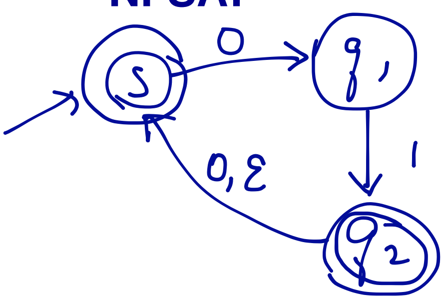
Q: How do we know if a *string* is *accepted* by an *NFSA*?

Check all possible paths and as long as one is accepted the string is accepted.

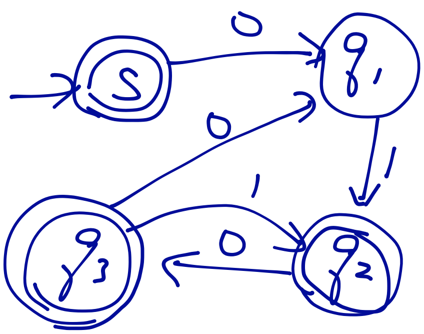
Examples of NFSA

Consider the strings that are *represented* by the *regular expression*: $(\mathbf{010} + \mathbf{01})^*$

NFSA:



DFSA:



Formally. An *NFSA*, is a machine $M = (Q, \Sigma, \delta, s, F)$ where

- each of Q, Σ, s, F are as for a *DFSA*.
- $\delta : Q \times (\Sigma \cup \{\epsilon\}) \to \mathcal{P}(Q)$ ($\mathcal{P}(Q)$ is a *set* of states).
- $\delta^* : Q \times \Sigma^* \to \mathcal{P}(Q)$.

Limitations of DFSA and NFSA

Q: Can *every* set of strings be *recognized* by a *DFSA*? an *NFSA*?

No.

Q: How much more *powerful* is an *NFSA* over a *DFSA*?

Not at all.

Detailed answers.

- Only strings representable by *regular expressions* can be *recognized* by an *NFSA* or *DFSA*.

- There exists an *algorithm* to *convert* between *deterministic* and *non-deterministic* machines.

Theorem. *If L is a regular language then the following are all equivalent:*

1. L is denoted by a *regular expression*

2. L is accepted by a *deterministic FSA*

3. L is accepted by a *non-deterministic FSA*

(See the course text for the proof.)

Closure Properties of FSA-accepted Languages

Q: What do we mean by *closure*?

if we perform an operation on members of a set, the new elements belong to the set.

Theorem Every *regular* language L is *closed* under *complementation, union, intersection, concatenation* and the *Kleene star* operation.

Q: *What does this mean?*

if L, L' are regular then so are $\overline{L}$, $L \cap L'$, $L \cup L'$ and LL' and L^*

Proof of $L \cup L'$.

- Let M be a *NFSA* that accepts L.
- Let M' be a *NFSA* that accepts L'.

Q: How can we *construct* $M_{\cup}$ that will accept *either* language?

add a new start state with ε transition to the start states of M, M'.

Proof of L^*.

Given M accepting L, how can we build a new *NFSA* to *accept* L^*?

from each accepting state add an ε transition to the start state and create a new start state to accept the ε string.

Regular Languages

Q: How can we prove that a *language* L is *regular*?

Q: How can we prove that a $\mathcal{L}$ is *not* regular?

- Any *FSA* has a *finite* number of *states*, say n.
- Therefore if L is *infinite*, then L has strings with $> n$ symbols.
- **Q:** *What does this imply about at least one state of the FSA?* is visited more than once which creates a cycle.
- Repeating this *cycle* an *arbitrary number* of times must yield another *string* in L.
- **Q:** *What does this mean?*
  On a string with $>n$ symbols, there must be a portion that is just a cycle being repeated.
- **Q:** *How does this help us?*

if we can create a string that is accepted but not in the language we have a contradiction.