## Lecture 4 Review

## Question \#1

- Find the groupings in the following K-Map

|  | $\overline{\mathbf{C}} \cdot \overline{\mathbf{D}}$ | $\overline{\mathbf{C}} \cdot \mathbf{D}$ | $\mathbf{C} \cdot \mathbf{D}$ | $\mathbf{C} \cdot \overline{\mathrm{D}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\overline{\mathbf{A}} \cdot \overline{\mathbf{B}}$ | 1 | 0 | X | 1 |
| $\overline{\mathbf{A}} \cdot \mathbf{B}$ | X | 0 | X | 1 |
| $\mathbf{A} \cdot \mathbf{B}$ | 1 | X | 1 | 1 |
| $\mathbf{A} \cdot \overline{\mathbf{B}}$ | 1 | X | X | X |

- Produce a logical equation for these groupings:

$$
A+\bar{D}
$$

## Question \#1: alternative

- Find the groupings in the following K-Map

|  | $\overline{\mathbf{C}} \cdot \overline{\mathrm{D}}$ | $\overline{\mathbf{C}} \cdot \mathbf{D}$ | $\mathbf{C} \cdot \mathbf{D}$ | $\mathbf{C} \cdot \overline{\mathrm{D}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\overline{\mathbf{A}} \cdot \overline{\mathbf{B}}$ | 1 | 0 | X | 1 |
| $\overline{\mathbf{A}} \cdot \mathbf{B}$ | X | 0 | X | 1 |
| $\mathbf{A} \cdot \mathbf{B}$ | 1 | X | 1 | 1 |
| $\mathbf{A} \cdot \overline{\mathbf{B}}$ | 1 | X | X | X |

- Produce a logical equation for these groupings:

$$
\bar{D}+C
$$

## Question \#2

- Complete the truth table


| S | R | $\mathrm{Q}_{\mathrm{T}}$ | $\overline{\mathrm{Q}}_{\mathrm{T}}$ | $\mathrm{Q}_{\mathrm{T}+1}$ | $\overline{\mathrm{Q}}_{\mathrm{T}+1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

$\leftarrow$ Hold
$\leftarrow$ Reset
$\leftarrow$ Set
$\leftarrow$ Forbidden

## Question \#2

- Complete the truth table


| $\mathbf{S}$ | $\mathbf{R}$ | $\mathbf{Q}_{\mathbf{T}}$ | $\overline{\mathbf{Q}}_{\mathbf{T}}$ | $\mathbf{Q}_{\mathbf{T + 1}}$ | $\overline{\mathbf{Q}}_{\mathbf{T + 1}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | X | X | 0 | 1 |
| 1 | 0 | X | X | 1 | 0 |
| 1 | 1 | X | X | 0 | 0 |

## Question \#3

- What are the output values from Q and $\overline{\mathrm{Q}}$ given the following inputs on $\mathrm{S}, \mathrm{R}$ and C ?


|  | $\mathbf{S}$ | $\mathbf{R}$ | $\mathbf{C}$ | $\mathbf{Q}$ | $\overline{\mathbf{Q}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 0 | 0 | 1 |  |  |
|  | 1 | 0 | 1 |  |  |
|  | 0 | 0 |  |  |  |
|  | 0 | 0 |  |  |  |
|  | 1 | 0 |  |  |  |
| 0 | 1 | 1 |  |  |  |

## Question \#3

- What are the output values from Q and $\overline{\mathrm{Q}}$ given the following inputs on S, R and C?


| Time | S | R | C | Q | $\overline{\mathbf{Q}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | 0 | 0 | 1 | ? | ? |
|  | 1 | 0 | 1 | 1 | 0 |
|  | 1 | 0 | 0 | 1 | 0 |
|  | 0 | 0 | 0 | 1 | 0 |
|  | 0 | 1 | 0 | 1 | 0 |
| $\downarrow$ | 0 | 1 | 1 | 0 | 1 |

## Question \#4 <br> 



Lecture 5 Review

## Question \#1

Assume we want to build a change machine

- We can add either \$0.05 or \$0.10 at a time
- We want to keep track of the current amount in the machine
- We can hold a maximum of \$0.50
- Draw the state diagram


## Question \#1b

- How many flipflops would you need to implement the following finite state machine
(FSM)?
- 11 states
- \# flip-flops =
$\left\lceil\log _{2}\right.$ (\# of states) $\rceil$

\# flip-flops = 4


## Question 2: Barcode Reader

- When scanning UPC barcodes, the laser scanner looks for black and white bars that
 indicate the start of the code.
- If black is read as a 1 and white is read as a 0, the start of the code (from either direction) has a 1010 pattern.

Can you create a state machine that detects this pattern?

## Step \#1: Draw state diagram



## Step \#2: State Table

- Write state table with Z
- Output Z is determined by the current state.
- Denotes Moore machine.
- Next step: allocate flipflops values to each state.
- How many flip-flops will we need for 5 states?

| Present <br> State | X | $\mathbf{z}$ | Next <br> State |
| :---: | :---: | :---: | :---: |
| A | 0 | 0 | A |
| A | 1 | 0 | B |
| B | 0 | 0 | C |
| B | 1 | 0 | B |
| C | 0 | 0 | A |
| C | 1 | 0 | D |
| D | 0 | 0 | E |
| D | 1 | 0 | B |
| E | 0 | 1 | A |
| E | 1 | 1 | D |

$$
\text { \# flip-flops = }\lceil\log (\# \text { of states) }\rceil
$$

## Step \#3: Flip-Flop Assignment

- 3 flip-flops needed here.
- Assign states carefully though!
- Can't simply do this:
$>\mathrm{A}=100>\mathrm{B}=011$
$>\mathrm{C}=010 \quad>\mathrm{D}=001$
$>E=000$
Why not?


## Step \#3: Flip-Flop Assignment

- Be careful of race conditions.
- Better solution:

$$
\begin{array}{ll}
>A=000 & >B=001 \\
>C=011 & >D=101 \\
>E=100 &
\end{array}
$$

- Still has race conditions $(C \rightarrow D, C \rightarrow A)$, but is safer.
- "Safer" is defined according to output behaviour.
- Sometimes, extra flip-flops are used for extra insurance.


## Step \#4: Redraw State Table

- From here, we can construct the K-maps for the state logic combinational circuit.
- Derive equations for each flip-flop value, given the previous values and the input X.

| $\mathbf{F}_{\mathbf{2}}$ | $\mathbf{F}_{\mathbf{1}}$ | $\mathbf{F}_{0}$ | $\mathbf{X}$ | $\mathbf{Z}$ | $\mathbf{F}_{\mathbf{2}}$ | $\mathbf{F}_{\mathbf{1}}$ | $\mathbf{F}_{0}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | 0 | 0 | 0 | 0 |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | 0 | 0 | 0 | 1 |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | 0 | 0 | 1 | 1 |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 0 | 0 | 1 |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | 0 | 0 | 0 | 0 |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 1 | 0 | 1 |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | 0 | 1 | 0 | 0 |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 0 | 0 | 1 |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | 0 | 0 | 0 |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 1 |

- Three equations total, plus one more for Z (trivial for Moore machines).


## Step 5: Circuit design

- Karnaugh map for $F_{2}$ :


$$
F_{2}=F_{1} X+F_{2} \bar{F}_{0} X+F_{2} F_{0} \bar{X}
$$

## Step 5: Circuit design

- Karnaugh map for $F_{1}$ :

|  | $\overline{\mathbf{F}}_{0} \cdot \overline{\mathbf{X}}$ | $\overline{\mathbf{F}}_{0} \cdot \mathbf{X}$ | $\mathbf{F}_{0} \cdot \mathbf{X}$ | $\mathbf{F}_{0} \cdot \overline{\mathbf{X}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\overline{\mathrm{~F}}_{2} \cdot \overline{\mathbf{F}}_{1}$ | 0 | 0 | 0 | 1 |
| $\overline{\mathbf{F}}_{2} \cdot \mathbf{F}_{1}$ | X | X | 0 | 0 |
| $\mathrm{~F}_{2} \cdot \mathbf{F}_{1}$ | X | X | X | X |
| $\mathbf{F}_{2} \cdot \overline{\mathbf{F}}_{1}$ | 0 | 0 | 0 | 0 |

$$
F_{1}=\bar{F}_{2} \bar{F}_{1} F_{0} \bar{X}
$$

## Step 5: Circuit design

- Karnaugh map for $\mathrm{F}_{0}$ :

|  | $\overline{\mathbf{F}}_{0} \cdot \overline{\mathbf{X}}$ | $\overline{\mathbf{F}}_{0} \cdot \mathbf{X}$ | $\mathbf{F}_{0} \cdot \mathbf{X}$ | $\mathrm{~F}_{0} \cdot \overline{\mathrm{X}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\overline{\mathrm{F}}_{2} \cdot \overline{\mathbf{F}}_{1}$ | 0 | 1 | 1 | 1 |
| $\overline{\mathrm{~F}}_{2} \cdot \mathrm{~F}_{1}$ | X | X | 1 | 0 |
| $\mathrm{~F}_{2} \cdot \mathrm{~F}_{1}$ | X | X | X | X |
| $\mathrm{F}_{2} \cdot \bar{F}_{1}$ | 0 | 1 | 1 | 0 |

$$
\mathrm{F}_{0}=\mathrm{X}+\overline{\mathrm{F}}_{2} \bar{F}_{1} \mathrm{~F}_{0}
$$

## Step 5: Circuit design

- Output value Z goes high based on the following output equation:

$$
z=F_{2} \bar{F}_{1} \bar{F}_{0}
$$

- Note: All of these equations would be different, given different flip-flop assignments!
- Practice alternate assignment for the midterm ©

